THE ROLE OF INTERNATIONAL MIGRATION OF UNSKILLED LABOR AND PARENTAL ABSENTEEISM IN THE HUMAN CAPITAL FORMATION OF CHILDREN LEFT BEHIND

By

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ABSTRACT

This study builds up a theoretical model to explore the combined effect of parental absenteeism due to emigration of unskilled labor and remittances on the economic growth in the source country through the channel of human capital accumulation of children left behind. The results of the study reveal that the international migration of unskilled adults constructively lowers down child labor in the economy. Moreover, this emigration is beneficial for the human capital formation of children left behind and economic growth in the source country only when the unskilled worker's relative wage is above a threshold level. In such a scenario, the positive effect of unskilled migration, in the form of an increase in the optimal proportion of time devoted to education by the child, overpowers the negative effect of parental absenteeism. The reverse holds true when the unskilled worker's relative wage is below a threshold level. This is because parental absenteeism emerges as a dominant force in this case and hampers human capital formation of children left behind, thus, adversely affecting the source country's growth rate.

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1 Introduction

The past several years have witnessed a major rise in the international migration of labor. The rising migration flows internationally have also been associated with a sound increase in remittances (Ratha et al. 2011). These remittances have proved to be an economic growth engine for a lot of developing countries (Ahn, 2004). Taking into account the central role played by migration and remittances in the economic growth of a country, this paper aims to focus on the international migration of unskilled labor. The objectives of this paper are two-fold. Firstly, the paper analyses the implications of international migration for child education in a scenario where the unskilled workers make the interdependent decisions of their migration and their children's education. Secondly, the paper analyses the impact of this unskilled migration on the economic growth of the source country. The study, hence, builds up on three fundamental premises:

1. Migration has important implications for child education.
2. Migration and child education happen to be joint decisions.
3. Education has major implications for economic growth.

A major chunk of the work done in this field is empirical in nature. The existing theoretical work discussing the interaction occurring between migration and schooling decisions, however, focuses on very different channels and transmission mechanisms. The only theoretical paper that is in line with our research question is Dessy & Rambeloma (2010). However, Dessy & Rambeloma (2010) fail to incorporate the fact that emigration also has a negative aspect attached with it. Our study goes much beyond the analysis done by Dessy & Rambeloma (2010), making a unique contribution to literature. By incorporating the aspect of parental absenteeism, this study offers a unique, coherent framework in the field of economics. There is no existing unifying framework that integrates all these aspects, namely endogenous decision making of migration and child education, remittances, parental absenteeism and economic growth. Moreover, adding the aspect of parental absenteeism is imperative for completely understanding the implications of international migration because absence of migrant parent might negatively affect the human capital accumulation of the child. Thus, net effect of international migration would be dependent upon whether the amount of remittances spent on education or parental absenteeism has a larger effect. Therefore, both factors play a role in the human capital accumulation of the child and not just remittance flows as found in Dessy & Rambeloma (2010). Thus, the analysis of migration and child education would be incomplete, or perhaps biased, without incorporating the essential component of parental absenteeism. Therefore, this study explores the combined effect of parental absenteeism due to emigration and remittances on the economic growth in the source country through the channel of human capital accumulation of children left behind.

The question of interest around which the study revolves does not only have great economic relevance but is also an attempt to provide a theoretical framework to formally illustrate the implications of the international migration patterns that we see today in the real world. World Bank's publication in 2011, Migration & Remittances Factbook, reveals that the number of people living outside the countries in which they were born has exceeded 215 million. From the period ranging from 2005-2010, the greatest influx of migrants was seen by United States. Moreover, the past several years have also witnessed a major rise in the migrant flows to the six GCC countries namely, Oman, Bahrain, Saudi Arabia, Kuwait, Qatar and the UAE, predominantly from East Asia as well as South Asia. Interestingly, the volume pertaining to the migration occurring between the developing countries i.e. South to South migration, comes out to be greater than the migration to the high-income OECD countries from South. The rising migration flows internationally have also been associated with a sound increase in remittances. Out of the worldwide remittances of \$440 billion, the amount of remittances flowing to the developing countries totaled \$325 billion in 2010. As compared to 2009, the developing countries have experienced a 6 percent increase in the remittance flows. Remarkably, the recorded remittances to the developing countries have been nearly thrice the level of foreign aid and just about the size of FDI flows in 2009. Even in the course of the recent financial crisis, the remittances flowing to the developing countries continued to remain resilient, showing merely a 5.5 percent decrease in 2009 while quickly recovering in 2010. On the other hand, FDI flows declined by 40 percent in 2009 (Ratha et al. 2011). Such trends and phenomena have triggered several economists to look at the socioeconomic implications of migration as well as remittances for the source country of migrants.

The remittances that the migrant workers send back home not only help in earning a significant amount of foreign exchange but also contribute considerably towards the economic improvement of the families of the migrants. Moreover, remittances have proved to be an economic growth engine for a lot of developing countries. Most South Asian governments consider outmigration as helpful in curtailing unemployment, reducing poverty, earning foreign exchange and contributing significantly towards the nation's economic development (Ahn, 2004).

This paper deals with various strands of economic literature shedding light on the aspects of migration, remittances, child labor, children's human capital accumulation, parental absenteeism and economic growth. The study would be reviewing the existing work on all these aspects, bringing out the inherent research void in this particular field, hence highlighting the motivation and contribution of this study.

There is a vast amount of empirical literature bringing out the impact of migration and/or remittances on the schooling of the migrant's children left behind in the source country. The empirical studies clearly bring to light the key role played by migration and remittances in fostering human capital accumulation of children belonging to the migrant households. These studies have been conducted in various countries ranging from Mexico, ranked at 71st place according to the Human Development Index (HDI) score, to the extreme poverty-stricken countries like Haiti, ranked at 168th place according to the HDI score, all bringing out the same message that if countries promote international migration then it can prove to be tremendously beneficial for the schooling of the migrant's children left behind in the source country. For instance, Hanson & Woodruff (2003), using data from the Mexican Census of 2000, performed an empirical analysis to examine how migrating to United States impacted the educational attainment of Mexican children. The findings of their study reveal that significantly greater schooling years are completed by the children belonging to households with an emigrant. This effect is particularly large for girls and they end up accumulating an additional 0.9 schooling years. The findings have been intuitively explained by the fact that the remittances sent back by the migrant member help in relaxing the credit constraints of the low-income households and therefore increase children's educational attainment.

While Hanson & Woodruff (2003) have explored the impact of migration on schooling years, Cox Edwards & Ureta (2003) have conducted an empirical analysis for El Salvador using data from 1997 to analyze migration's influence on the likelihood of dropping out from school. Employing the Cox proportional-hazards regression model, the findings of their analysis reveal that remittances largely and significantly enhance school retention and reduce the hazard of children to drop out from school. As compared to the effect exerted by other income, the influence of remittances is 10 times greater in the urban areas and 2.6 times greater in the rural areas. Another interesting finding of this study is that merely the fact that remittances are being received, irrespective of the amount of remittances, reduces the likelihood of dropping out from school in the rural areas of El Salvador. The study brings out an important policy implication suggesting that if school attendance is subsidized especially in the poor areas then it can largely affect school retention despite the low schooling levels of parents. This is particularly important for poor countries such as El Salvador where the budget constraint of families plays a major role in shaping the family's decision to send their children to school.

Although empirical studies have illustrated the role of remittances in children's educational attainment, Dorantes et al. (2010) have made an attempt to distinguish between the remittance effect and migration effect. Social, economic as well as environmental indicators have consistently ranked Haiti among the extremely disadvantaged countries of Western Hemisphere, having high poverty levels. However, the proportion of remittance receipts in the GDP of Haiti has increased from 5 percent in 1996 to a figure of 21.5 percent in 2006. This clearly shows that for Haiti remittances comprise of a significant magnitude and might play a key role to raise the living standards of its people. Therefore, Dorantes et al. (2010) conducted an empirical study, using data for 2000 and 2002, to evaluate the impact exerted by remittances on the children's educational attainment in the origin communities of emigrants. The authors have separately looked at migration effect and remittance effect. The findings of the study reveal that remittances contribute towards ameliorating the negative and disruptive effects exerted by the out-migration on the schooling of children and hence, contribute towards human capital accumulation even in an extreme poverty-stricken country, like Haiti.

Not only do migration and remittances help the children to acquire greater years of education, enhance school retention, reduce the likelihood of dropping out from school and increase school enrolment but also help in reducing child labor. A lot of studies have empirically shown the dual role of remittances in enhancing school enrolment on one hand while reducing child labor on the other in El Salvador (Acosta, 2006), Pakistan (Mansuri, 2006), Ecuador (Calero et al., 2007) and Philippines (Yang, 2008) amongst many others. To empirically demonstrate this dual role of remittances, Acosta (2006) conducted an empirical study to analyze how the international remittances affect the spending decisions of households. Remittances increase household budgets and lessen liquidity constraints, making the investment in the human capital of children affordable. This is very important from the growth perspective of developing countries. The findings of the study show that the girls and boys (below the age of 14) belonging to the recipient households have a greater likelihood of enrolling in school as compared to those belonging to the non-recipient households. Moreover, it was also shown that remittances negatively affect child labor as well as the labor supply of adult females while not affecting the labor supply of adult males. The study, while spelling out the twofold role of remittances in child education and child work decisions, also brings to light the gender differences inherent in the way remittances are used within households.

This twin role of remittances in child educational attainment and child labor has also been specifically analyzed in the context of temporary emigration pertaining to low skilled labor by Mansuri (2006). The author uses 2001-02 data for rural Pakistan to empirically explore the impact on investment in children's schooling when low skilled labor temporarily migrates. The results of the study highlight large positive effects exerted by temporary migration on the accumulation of human capital. Additionally, these gains appeared to be much larger for girls, thereby substantially reducing gender inequalities inherent in the access to the educational system. The girls' dropout rate declined by 55%, while for boys there was a 44% decline. Girls belonging to the migrant households had an additional 1.5 schooling years as compared to those belonging to the non-migrant households while the boys had one grade more. Migration also strongly dampened down child labor activity, significantly reducing the number of hours worked. There was an overall reduction in the days worked by 66% (from 27 days to 10 days for boys and from 27 days to 9 days for girls). This study brings to light an important policy implication when the author suggests how opening up of the international labor market for the temporary emigration of the low skilled labor from the developing countries can potentially enhance the accumulation of human capital by poor people. The huge amount of remittances sent back by the migrants underscores the importance of this issue.

Moreover, the role of remittances in enhancing school enrolment and reducing child labor has also been analyzed by Calero et al. (2007) by using 2005-06 data for Ecuador. The authors investigate how remittances stimulate investments in human capital by relaxing the resource constraints along with facilitating the households to smooth out consumption by decreasing vulnerability to the economic shocks. The findings of the study highlight an increase in school enrolment along with a decrease in child labor due to remittances, particularly for girls as well as in the rural areas. The authors especially draw attention to the fact that the rural areas, in particular, are vulnerable to covariate risks plus liquidity constraints which affect the investments in children's human capital. Remittances serve as an important coping mechanism when households experience such shocks by helping them to finance their children's education.

Furthermore, the double effect of remittances in augmenting child education and reducing child labor was more clearly illustrated in Philippines. Filipino workers were working overseas in various foreign countries where sudden changes in the exchange rate were experienced owing to the financial crisis taking place in Asia in 1997. Yang (2008), taking advantage of the currency crisis and using it as a natural experiment, examined the responses of the Philippine households as a result of the economic shocks experienced by the overseas family members. The household remittances being received from the overseas members reported an increase as the migrant's currency appreciated against Philippine peso. These favorable income shocks enhanced the accumulation of human capital in the origin households of the migrants. Resultantly, child schooling as well as educational expenditures increased and child labor declined. Millions of families in the developing countries depend upon the financial support received from the family members who are working overseas. The author draws attention to the fact that policies of the developed countries which influence migrant workers could have an effect on the households living in the poor countries. The findings of the study can be applied to predict the effect of a reduction in the remittance sending costs because such reductions would effectively mean an exchange rate improvement faced by the remittance senders. In a broader picture, the author suggests that if the rich countries pursue policies that expand employment opportunities in favor of the overseas workers then it can stimulate investment in human capital in the poor-country households. Moreover, policies which allow the workers who are currently undocumented in obtaining legal work permits would expand the migrants' earning opportunities and hence, enhance investment in human capital in the origin households of the migrants. On the other hand, if enforcements against the illegal immigrants are increased or temporary work permits for the overseas migrants are eliminated then it would reduce the earning opportunities for migrants and hence, discourage such investments by the origin households.

The empirical studies cited so far have been conducted in specific countries. Ebeke (2012), however, presents an all-encompassing analysis using data for eighty-two developing countries to examine the underlying relationship between remittances and the child labor prevalence. The findings of the study reveal that remittances help in reducing the child labor incidence in the developing countries where financial development levels are low and macroeconomic instability is high. The author explains two main mechanisms by way of which migration along with remittances could reduce child labor as regards developing countries. Firstly, in financially constrained households, remittances act as an alternative funding source making the supply of labor by children less likely. Secondly, remittances help in reducing child labor by acting as a cushion against the income shocks which contributes towards lowering down the child labor sensitivity to these macroeconomic shocks. The econometric results particularly highlight that (i) remittances significantly decrease the child labor prevalence in the situation where financial constraints are high, and (ii) remittances help in dampening the damaging influences exerted on child labor prevalence by the volatility in income growth. The results of the paper strongly suggest that each and every strategy which helps in facilitating remittance inflows is critically important for the formation of human capital.

Where on one hand there is a massive volume of empirical literature underlining the significance of migration and remittances for human capital accumulation of children belonging to the migrant households, there is a dearth of theoretical work on this precise channel. However, a lot of theoretical work linking migration and education exists, albeit focusing on very different transmission mechanisms. For instance, Mountford (1997) theoretically illustrates that brain drain (emigration of skilled labor) might enhance a developing country's productivity in case of endogenous educational decisions and uncertainty regarding successful emigration. This is because the probability to migrate to a country offering higher wages would enhance the returns accruing to education. This would resultantly increase human capital accumulation, thereby outweighing the negative effects exerted by brain drain. In this way brain drain could prove to be beneficial for the source country's economic growth. Likewise, Vidal (1998) shows the existence of a positive correlation between the emigration probability and the human capital level in the long run in the country of origin. In a similar vein, Beine et al. (2001) theoretically examined the influence exerted by the prospect to migrate on the human capital accumulation in a framework incorporating uncertainty. The authors explain that in poor economies the growth potential is inadequate, having low returns to education. This results in limited incentives of acquiring education, which happens to be an economic growth engine. Nonetheless, the world values education. Therefore, if people are allowed to migrate from this poor economy, this would result in an increase in the proportion of its educated population. However, there is uncertainty regarding the opportunity to migrate in future. As a result, some people would make educational investments merely for the reason of the opportunity to migrate, but such an opportunity might not materialize. Consequently, only a few educated people would migrate abroad, thereby increasing the remaining population's average educational level. Thus, by using this channel the authors highlight how brain drain could be beneficial for a country with migration possibilities.

Moreover, Stark & Wang (2002) theoretically demonstrate how migration prospects could be harnessed for inducing the individuals to accumulate a human capital level that is socially desirable. The authors underscore the role of greater prospective returns for human capital existing in foreign countries that impinges on the decision to accumulate human capital at home. The theoretical findings of the study reveal that migration prospects can act as a substitute for public subsidies' provision, and hence generate a socially preferable human capital level. Furthermore, Mayr & Peri, (2009) bring to light the favorable schooling incentive effects induced by the prospect of migrating to countries where education entails higher returns. The findings of their theoretical analysis show that the prospect of migrating temporarily to such countries generates almost the same incentive effects such as those generated by the prospect of migrating permanently. This is because the individuals undertaking migration and return decisions make greater investments in schooling because education would entail higher returns in the foreign country and also once they return.

All of the theoretical studies cited above emphasize on how agents themselves accumulate human capital in light of their own migration opportunities. There is, yet, another branch of theoretical work that focuses on a different transmission mechanism. Theoretical modelling in this field deals with how the prospect of their children emigrating in future induces the parents to invest in their children's education. For instance, according to Chen (2006), parents make fertility as well as education decisions and their children have the opportunity of migrating to some foreign country. Since the fertility decisions have been internalized, parents are faced with a trade-off concerning their children's quality as well as quantity when the migration probability changes. This quality-quantity trade-off affects the accumulation of human capital by the children, thus affecting economic growth. Since, Chen (2006) has not incorporated remittances, Marchiori et al. (2010) have proposed a theoretical model where prospect of children's emigration and remittances both contribute towards shaping the fertility and educational decisions of households, thus, influencing the formation of human capital in the source country. In this study, it has been assumed that only the highly skilled children can migrate and that too with a specific probability. The theoretical findings of the study show that the possibility of this emigration and the higher amount of remittances that these children would send back to their parents encourages both low and high skilled parents to invest in the higher education of their children.

Our study, however, focuses on a very different transmission mechanism than those discussed in the existing theoretical literature. In our study, the emigration of parents and the remittances that they send back influence economic growth in the source country through the channel of human capital accumulation of their children. The only two theoretical papers that focus on this channel are Dessy & Rambeloma (2009) and Dessy & Rambeloma (2010). The endogenous model of economic growth employed by Dessy & Rambeloma (2009) lays its foundations on the fact that the skilled and less skilled workers are forward-looking and therefore make joint decisions to emigrate and to send back remittances to the children left behind in the country of their origin. Their study revolves around the role of immigration policy adopted by the host country in the economic growth of the source country. They discuss cases where the immigration policy favors skilled workers as well as the less skilled migrants. However, in our paper, we do not incorporate the aspect of immigration policy and only focus on the individual decisions employed by the workers who are not skilled at all, i.e., the unskilled workers. Since Dessy & Rambeloma (2010) focus explicitly on unskilled migration, our study makes use of their model as the base model.

Our study particularly focuses on the migration of unskilled workers so as to theoretically model the impact of the recent migration trends in the developing countries. The unskilled workers from the South not just migrate to the richer economies of the North but a lot of them also migrate to other countries in the South as well. The increasing volume of this South to South migration is becoming a special feature of the emigration taking place in the developing countries. In 2010, 74 million migrants from the developing countries were living in other developing countries which was greater than the South-North migration (Ratha et al. 2011).

As pointed out by Hicks in his famous book The Theory of Wages written way back in 1932, wage differences are the major reason behind migration (Hicks, 1932). In our paper, unskilled workers emigrate to the economy where they find positive wage differentials, be it in the North or South. According to Fuest & Thum (2001), these favorable wage differentials are due to trade unions as well as minimum wages keeping the wages for the unskilled workers higher than the market clearing level. Unionized labor markets make those countries really attractive to the unskilled migrants because of the additional amount of rents which can possibly be earned. Moreover, unionization and higher level of unemployment insurance benefits create such differences in real income that even the mobility costs as well as tax policies do not become an obstacle in the decision of the unskilled workers to emigrate (Zimmermann et al. 1994). Furthermore, most of the international migration involves young workers migrating from countries in which they are in abundance to the countries in which they are scarce (Ortega & Peri 2009). This scarcity creates favorable wage differentials for unskilled young workers.

During the middle of the 20th century, chiefly the professionally qualified people from the South Asian countries were migrating to the developed countries. However, the increasing oil prices in the 1970s created a massive demand for various labor categories in the Middle Eastern oil-producing countries. During this time, workers from the South Asian countries migrated to UAE, Saudi Arabia, Iraq, Qatar, Libya, Oman and Kuwait. From the mid-1980s, this migration had expanded to the South East Asian and East Asian countries namely Singapore, Korea and Malaysia which were facing a severe shortage of workers willing to accept the 3D jobs- dirty, degraded as well as dangerous. The major chunk of migrants from the South Asian countries comprise of the unskilled workers who willingly take up these 3D jobs as opposed to the domestic workers (Ahn, 2004). All of these conditions have created demand for the unskilled workers, producing positive wage differentials. Therefore, our paper specifically focuses on the migration of unskilled workers, making the study particularly applicable to the developing countries. Besides, several papers have empirically tested the implications of international migration but this issue has not been taken up in the theoretical field. Hence, there is a need to design a proper theoretical model that clearly spells out the parameters and conditions under which the international migration of unskilled labor is beneficial for the economic growth of the source country. Our study effectively caters to this need.

In our study, the unskilled workers from the source country might migrate to the destination country to avail the wage premiums associated with the migration of unskilled workers. The unskilled parent who emigrates to the destination country leaves his/her child behind in the source country. The child then becomes a recipient of the parental remittances. In such an environment, the parent undertakes three crucial decisions. The parent jointly decides the proportion of his/her total time to be spent in the destination country, the amount of remittances to be sent back to the child dependent, and the time his/her child should devote to education. Parental remittances become a source to finance the consumption of the child. However, for the purpose of financing his consumption, the child might have to supplement these remittances by engaging in child labor. Although the very basic setup is taken from Dessy & Rambeloma (2010), our study is different from their study in a number of ways, making a unique contribution to literature. Firstly, our study endogeneises the migration decision as opposed to the stochastic setup used by Dessy & Rambeloma (2010), where parents were randomly drawn and given the right of emigrating to the richer economy. We however would treat migration and child education as a joint decision-making process. Empirical literature clearly acknowledges migration and child education as interdependent decisions and takes steps to tackle this endogeneity issue. For instance, in the study by Hanson & Woodruff (2003), the migration behavior has been treated as endogenous because similar factors influence the migration decision as well as children's schooling. In order to overcome the endogeneity issue, the authors instrumented household migration by using data for the historical patterns of migration in Mexico. Similarly, in Acosta (2006), by instrumenting remittance receipts by village along with household networks, the author gives robust estimates particularly dealing with the methodological concerns arising from selection as well as endogeneity issues. Moreover, Mansuri (2006) also deals with the econometric challenge of endogeneity regarding the decision to migrate by using the instrumental variable approach. This is because there is no random assignment of migration to the households and many similar characteristics influence both the migration choice and the ability of households to make educational investments. Therefore, the author instruments migration by using the migration rates prevailing at the level of villages but varying at the household level.

Furthermore, Calero et al. (2009), while analyzing the role of remittances in enhancing school enrolment and reducing child labor, use the identification technique of instrumental variables to instrument for remittances. The remittances that households receive are possibly endogenous to the decisions regarding human capital and supply of child labor. This is because income shocks influence investments in human capital and at the same time remittances are simultaneously adjusted by these shocks in order to reduce the volatility in income. Therefore, the authors used the instrumental variable technique by exploiting the information on countries that are source of remittances. Moreover, they also used the regional variation pertaining to the bank offices' availability. These bank offices are the formal channels to send remittances. Both of the instruments employed reflect the transfer costs involved, hence partly determining the frequency along with the volume of the funds transferred. Moreover, Yang (2008) wisely tackled the endogeneity issues inherent in migrant earnings and human capital investments made by the migrants' households by taking advantage of the currency crisis and using it as a natural experiment. In this way, the shocks to the exchange rate reflected an exogenous variation in the amount of remittances received, hence overcoming the endogeneity issue. Furthermore, the endogeneity issue has also been addressed by Dorantes et al. (2010) where the authors use the instrumental variable approach. The remittances that households receive have been instrumented by using two variables. The first instrument was the weekly worker earnings of the workers living in United States, being similar to the possible Haitian remitters. The second variable traces unemployment in the geographic areas where there is a likelihood of the household having migrant networks. The authors found sufficient correlation of these instruments with the remittances that households receive, hence addressing the endogeneity issue. Similarly, Ebeke (2012) also controlled for the endogeneity issues regarding remittances, migration as well as financial development using the instrumental variable approach. The authors instrumented remittance receipts by the costs involved in sending back US $200. Also, the author used dummy variable that indicated presence of a dual system of exchange rate to instrument remittance receipts. Adult migration to the OECD countries was instrumented by the country's coastal area and the existing distance between OECD countries and each of the developing countries.

All of these studies highlight the fact that empirical studies do recognize the inherent endogeneity in migration and child education decisions and resort to various steps to overcome this methodological concern. In addition to that, theoretical papers have also treated migration as endogenous when people incorporate the probability to migrate while undertaking educational investment decisions (Mountford, 1997; Beine et al., 2001; Stark & Wang, 2002; Beine et al., 2008; Mayr & Peri, 2009;Chen, 2006; Marchiori et al., 2010). Therefore, there is a need to treat migration and child education as a joint decision-making process. Our study caters to this issue, where the unskilled workers make the interdependent decisions of their migration and their children's education, contributing to the work of Dessy & Rambeloma (2010).

Moreover, Dessy & Rambeloma (2010) do not have any human capital accumulation function in their setup. So the second way in which our study is different from their study is that we will be introducing human capital accumulation technology. In this way, the study enables us to see the impact of international migration of unskilled labor on the accumulation of human capital by children. The remittances sent back by the migrants help to finance the consumption needs of the children, allowing them to postpone their participation in the market for child labor, hence extending their enrolment in school. Therefore, by shifting the use of children's time towards education and away from working, remittances play a major role to promote school enrolment and hence, curb child labor.

The third way in which our study is different from the base study i.e. Dessy & Rambeloma (2010) is that we will be introducing human capital accumulation technology as a function of parental absenteeism. The human capital of children is adversely affected by parental migration because of three main reasons. Firstly, the households that experience migration are similar to disrupted families that exerts negative psychological effects on children. This consequently has a bearing on the educational performance of children (Kandel & Kao, 2001; Bennett et al., 2012). Secondly, owing to migration, rearing along with housework responsibilities are placed on the children who are left behind which affects the children's time allocated to school-related work. Moreover, the children who are left behind have to assume the role of their parents as the breadwinner and provider and hence join the workforce at an earlier age, taking on a parent figure role for the younger siblings (Booth & Tamura, 2009; McKenzie & Rapoport, 2010). Thirdly, if children develop a perception that their parents can earn higher wages as a result of migrating to a foreign country and working in the unskilled jobs then it greatly reduces the incentives of children to attain higher educational levels. For instance, migration is seen as an alternate route of achieving success in economic sphere without attaining higher educational levels (Kandel & Kao, 2001). Therefore, both factors namely remittances and parental absenteeism play a role in the human capital accumulation of the child. Where on one hand remittances positively affect the human capital accumulation process of the child, parental absenteeism has a negative effect. Thus, the analysis of migration and child education would be incomplete, or perhaps biased, without incorporating the essential component of parental absenteeism. For that reason, our study caters to this concern and studies the implications of the international migration of unskilled labor from both positive and negative aspects, providing an all-encompassing analysis of migration.

Finally, our study looks at the impact exerted by migration along with remittances on the economic growth of the source country. Dessy & Rambeloma (2010) do not analyze the growth implications of migration. However, analyzing the growth implications is important because education is an economic growth engine. There exists a rich volume of theoretical literature illustrating the chief role played by education in realizing sustained economic growth (Nelson & Phelps, 1966; Azariadis & Drazen, 1990; Mankiw, Romer and Weil, 1992; Redding, 1996). Moreover, the human capital theory proposed by Lucas (1988) lays emphasis on the fact that the primary element driving economic growth is human capital accumulation. Additionally, countries differ in their economic growth rates because of the differences pertaining to the rates of accumulating human capital.

Not only does international migration contribute towards augmenting human capital accumulation by the emigrants' children but it also promotes economic growth in the migrant's country of origin. There is a considerable amount of literature illustrating the impact of migration on economic growth. Ziesemer (2012) has conducted an empirical analysis to investigate the effect exerted by migration and remittances on the economic growth of 52 developing countries having per capita incomes less than \$1200. The results of the study highlight that remittances have a direct positive effect on GDP per capita's levels as well as growth rates in these poor countries. Firstly, the results show that remittances majorly have a role in increasing savings, which enhances emigration, consequently reducing the growth of labor force and hence enhancing GDP per capita's growth. Secondly, remittances have a role in enhancing education related variables directly as well as indirectly through savings. Improving literacy helps to reduce the growth of labor force and increase investment, consequently enhancing GDP per capita's growth. The author suggests that it would be bad to stop migration for these poor countries unless migration has a powerful skill bias.

Where Ziesemer (2012) has drawn on an empirical approach to bring out the role of migration in the economic growth of the source country, Chen (2006) has resorted to a theoretical method to highlight this linkage. According to Chen (2006), human capital plays a role when investigating the impact exerted by migration on the source country's economic growth. The paper highlights the crucial dependence of economic growth on international migration because the migration probability would influence fertility decisions as well as school expenditures. The findings of the study reveal that if the probability of the low-skilled workers to emigrate is greater than the optimal migration probability, then with a lot of low-skilled workers migrating to the foreign countries would result in a rise in the domestic country's economic growth. This is because the school expenditures would rise and the percentage of workers with low skills would fall in the labor markets. Subsequently, this would enhance the accumulation of human capital by the low-skilled parents' children under both the public as well as private education regimes, resulting in a major "brain gain". The study brings to light some thought-provoking policy implications. The author suggests that if the source country's government aims to enhance economic growth then some restrictions need to be placed on the international migration of the high-skilled workforce. Relaxing the restrictions concerning the high-skilled workers' emigration could prove to be damaging to the source country's economic growth over the long term.

This uncovers an important issue regarding international migration. Not all types of emigration are beneficial for the country of origin. In the 1970s, several prestigious economists were of the view that the international migration of skilled labor had detrimental effects on the source country. Such emigration was seen as a mere zero-sum game making the already rich countries richer while causing the poor ones to become poorer (Rapoport 2002). According to Bhagwati & Hamada (1974), the emigration of skilled labor induces strong negative externalities on the sending countries. They further stress upon the fact that the situation is particularly worrisome for underdeveloped countries where those who are left behind are in a loss due to the emigration of doctors and extraordinarily gifted academics.

Moreover, Docquier (2006) has also explained how the issue of brain drain could prove to be a threat particularly for the developing nations. He highlights that a lot of studies have emphasized how huge gains have been generated for the migrants' families as well as the source countries due to the migration of the unskilled labor. Since the constraints pertaining to labor market are relaxed at the origin and large remittance flows are induced, migration of the unskilled labor force should undoubtedly be viewed as an obvious and explicit part of rich world's development policy. On the other hand, the international migration of skilled labor deprives a developing country of its scarcest resource, human capital. Therefore, skilled migration impoverishes the source countries while the receiving countries end up making handsome profits. When the extremely talented workforce emigrates then it reduces the labor force's average human capital level. Ceteris paribus, such a reduction in the human capital directly exerts a negative effect on GDP per capita. In the long run, this decrease in the human capital has a serious effect on the capacity of the country to innovate as well as to adopt modern technology. Therefore, brain drain negatively affects total factor productivity along with increasing distance to the production frontier.

Not only does skilled emigration negatively affect the source country's economic growth but the skilled labor also sends back less remittances as compared to the unskilled workers. Niimi et al. (2010) conducted an empirical analysis covering 82 countries. The results of their study highlight that remittances decrease with a rise in the overall educational level of the migrants. This is because the skilled migrants' families are generally better off and demand remittances less as compared to the poorer families. Moreover, the legal status of skilled migrants in the destination country is more secure so it enables them to take their families with them. All these factors combined act behind reducing the incentives of sending remittances. The sending countries have been highly concerned about the negative impacts of brain drain and hence this provides them with another reason due to which they would prefer the emigration of unskilled labor as opposed to the skilled labor emigration.

Likewise, Rapoport (2002) also sheds light on the remittance issue associated with brain drain. According to him, household surveys have shown that remittances sent back home by the educated migrants tend to be lower than those sent by the uneducated migrants. Though the earning potential of the skilled migrants is much higher but they migrate permanently with their families and, hence, have a tendency of remitting relatively less as compared to the unskilled migrants. Faini (2007) has conducted an empirical analysis on this issue for a large developing countries' panel. First the author builds up a simple theoretical model indicating that the skilled migrants have lower propensity of remitting home from the given earnings flow abroad. Then empirical investigation revealed considerable evidence of the association of brain drain with smaller remittance flows. The author explains the intuition behind the results by giving the reason that the skilled migrants have a greater likelihood of spending longer periods of time in the foreign country as well as reuniting with their family in the destination country. Both of these factors result in a smaller remittance flow from the skilled migrants. Moreover, Lucas & Stark (1985) also highlight that there is a tendency for the remittance flows to decline as the duration of the stay of the migrants increases. This is because the willingness of reunifying with their family members in the destination country as well as facing lesser constraints enables the skilled migrants to spend longer time abroad. Furthermore, in order to explore the reasons determining the different amounts of remittances received by developing countries, Richard & Adams (2009) conducted an empirical analysis using data on several variables, namely poverty, migrants' skill composition, interest as well as exchange rates. The results of the study highlight that migrants' skill composition does matter as regards remittance determination. It was shown that the developing countries which are exporting a greater proportion of highly skilled labor end up getting lower remittance receipts per capita as compared to countries which are exporting a higher proportion of people with low skills.

Thus, it is worthwhile to analyze the economy-wide impact of unskilled migration. The results of our study reveal that the decision of the unskilled workers to migrate is determined by the wages that unskilled workers face abroad and at home as well as the wage that their children earn in the source country. When the absolute difference between the unskilled worker's wage in the destination country and the source country grows, it motivates them to devote larger proportion of their time in the destination country to take advantage of the increasing wage spread. Besides, it is not just the unskilled worker's wage that affects their migration decision but, interestingly, the child's wage also influences unskilled migration. Higher child wage reduces the need to supplement the income earned by the child with remittances and encourages the parents to spend greater time in the country of origin, thereby discouraging migration.

The results of this study also highlight the role of the wage earned by unskilled workers in the destination country as well as the wage earned by their children in determining the amount of remittances. According to the results, an increase in the unskilled worker's wage in the destination country increases the amount of remittances which in turn increases the children's time spent on education by relaxing financial constraints. On the other hand, an increase in the child's wage reduces the remittance flows, consequently reducing the children's time spent on education.

The results of our study also bring to light the factors affecting child labor in the economy. According to the results, the international migration of unskilled adults and a rise in the unskilled worker's relative wage constructively lowers down the economy-wide child labor incidence while an increase in the productivity of children increases child labor in the economy. Moreover, the international migration of unskilled workers is beneficial for the human capital formation of children left behind and economic growth in the source country only when the wages offered in the destination country are sufficiently larger than the wages that are being offered to the unskilled workers in the source country. In such a scenario, the positive effect of unskilled migration, in the form of an increase in the optimal proportion of time devoted to education by the child, overpowers the negative effect of parental absenteeism. On the other hand, when the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country is below a threshold level then the migration of unskilled workers adversely affects the growth rate. This is because parental absenteeism emerges as a dominant force in this case and hampers human capital formation of children left behind.

Moreover, the size of the threshold value of the unskilled worker's relative wage is determined by the relative importance of the parental time spent in the source country and the child's time spent on education in the determination of the human capital of the future generation. If the human capital of the future generation is very sensitive to parental absenteeism then this increases the size of the threshold value. As a result, the unskilled worker's relative wage has to be greater than a larger threshold value to overcome parental absenteeism so as to produce an overall positive impact on children's human capital and economic growth. On the other hand, if the parental time spent in the source country is relatively less important than the child's time spent on education in determining the human capital of the future generation, then this implies that children's human capital is less sensitive to parental absenteeism, reducing the size of the threshold value. Consequently, the unskilled worker's relative wage has to be greater than a smaller threshold value to overpower the negative influence of parental absenteeism so as to exert an overall positive impact on children's human capital and economic growth. Finally, if the parental time spent in the source country and the child's time spent on education are equally important in determining the human capital of the future generation, then the international migration of unskilled workers is beneficial for the human capital formation of children left behind and economic growth only when the unskilled workers can earn more than double the wages that they can earn while staying in the source country. The results of the study bring to light important policy implications particularly for developing countries. The results imply that caution must be exercised by countries using migration of unskilled workers as a tool to promote economic growth in the country of origin. Countries can use it to wisely devise policies aimed at promoting international migration of unskilled workers so as to foster economic growth in the source country.

The organization of the rest of the paper is as follows: section 2 describes the model setup, section 3 solves the decision problems of parents, section 4 conducts a comparative static analysis, section 5 looks at the impact of unskilled migration on human capital formation of children and economic growth while section 6 presents the concluding remarks and policy recommendations.

2 Methdodolgical Framework: Model Setup

This section describes the methodological framework of the study. The model assumes a world having two economies, the destination country and the source country. The source country contains both the skilled and the unskilled sectors of labor. The total population of the source country is N_t which is split up into the skilled adults L_t^S and the unskilled adults L_t^U as shown:

$$N_t = L_t^S + L_t^U \tag{2.1}$$

However, in this setup, only the unskilled labor is assumed to migrate to the destination country and the skilled labor does not migrate[1]. There are wage premiums associated with the migration

[1] This has been assumed just to focus on the implications of unskilled migration. See Dessy & Rambeloma (2009) and Camacho & Shen (2010) for skilled migration.

of unskilled workers:

$$w_{UD} = \alpha w_{US} \tag{2.2}$$

where w_{UD} denotes the unskilled worker's wage in the destination country, w_{US} represents the unskilled worker's wage in the source country, and α is the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country ($\alpha > 1$). Therefore, unskilled workers from the source country might migrate to the destination country. In the unskilled sector of the source country, households comprise of parents (unskilled adults) and children. In this setup, one period is left for each unskilled adult to live. Every household has a unique child who would live for two periods. Each child has a time endowment of one unit. The allocation of the time endowment of a child is between education and work.

Parents migrate to the destination country, leaving their children in the source country. These children then become recipients of the parental remittances. In such an environment, the parents undertake three crucial decisions. The parents decide (i) the proportion of their total time to be spent in the destination country; (ii) the amount of remittances to be sent back to dependent child, and (iii) the time their child should devote to education. Parental remittances become a source to finance the consumption of the child. However, for the purpose of financing his/her consumption, the child might have to supplement these remittances by engaging in child labor.

2.1 Preferences and the Budget Constraints

The model used in this study suits the case of temporary migration better. Moreover, the household concept employed in this model is more suitable for the extended family of the migrant considering temporary migration. A lot of policy reports have identified the role played by the members of the extended family in partially bearing the child-bearing responsibility during the absence of migrant parents (Camacho & Shen 2010).

In this setup, parents jointly decide about the proportion of their total time to be spent in the destination country (δ),the sum of money, θ_t, to be remitted to child in the source country, as well as the proportion of time, e_t, that this child would devote to education. All the parents hold identical preferences as regards their own-consumption (c_t), the consumption of their child (c_t^k), and the level of human capital of child (H_{t+1}). These preferences can be specified in a utility function that is additively separable:

$$U_t = \ln(c_t) + \gamma\,[\ln\, c_t^k + \beta \ln\, H_{t+1}] \tag{2.1.1}$$

where U_t denotes the utility function of the household in time period t, $\gamma > 0$ represents the parental altruism level and $\beta \,\epsilon\, (0, 1)$ is the discounting factor. In this model, education emerges as the key mechanism for accumulating human capital. The child who is spending a fraction, e_t, of his/her total time endowment in receiving education would end up accumulating the human capital level, H_{t+1}, according to the following human capital accumulation technology:

$$H_{t+1} = \max\,\{\underline{H}, \lambda H_t e_t^{\sigma_1}(1-\delta)^{\sigma_2}\}, \qquad 0 < \sigma_1, \sigma_2 < 1, \qquad (\sigma_1 + \sigma_2) < 1 \tag{2.1.2}$$

where $\lambda > 1$ denotes the productivity parameter and H_t is the teachers' mean human capital, where hiring of teachers takes place from the skilled sector. Adults in a household have time endowment of one unit. δ denotes the proportion of total time spent by the household adults in the destination country while $(1 - \delta)$ shows the proportion of time that the unskilled adults spend in the source country. The human capital accumulation technology takes on this form so as to avoid the condition where H_{t+1} takes on a value of zero. If the unskilled adults in every household spend their entire time in the destination country (i.e., if $\delta = 1$) or if the child does not devote any time to education (i.e., if $e_t = 0$), then the level of human capital of child does not drop to zero but takes on the value of a minimum level of human capital $\underline{\text{H}}$. $\underline{\text{H}}$ is very low and close to zero. It depicts a situation where children can still accumulate some human capital through informal learning outside the formal schooling even if they do not attend formal schools.

The human capital accumulation technology in our study incorporates the aspect of parental absenteeism. Adding the aspect of parental absenteeism is imperative for completely understanding the implications of international migration because absence of migrant parent might negatively affect the human capital accumulation of the child. Thus, net effect of international migration would be dependent upon whether the amount of remittances spent on education or parental absenteeism has a larger effect. Therefore, both factors play a role in the human capital accumulation of the child and not just remittance flows as found in Dessy & Rambeloma (2010).

Thus, the analysis of migration and child education would be incomplete, or perhaps biased, without incorporating the essential component of parental absenteeism. The term $(1-\delta)$ shows the negative impact of parental absence because of migration (Camacho & Shen 2010). By incorporating the aspect of parental absenteeism, our study caters to both the positive and the negative aspects of migration, hence, presenting a holistic analysis of the international migration of unskilled labor.

The human capital accumulation technology exhibits diminishing marginal returns to the child's time spent on education. Such diminishing marginal returns to the time invested in human capital are also supported by Azariadis & Drazen (1990) and Redding (1996). Moreover, our human capital technology also exhibits diminishing marginal returns to the parental time spent in the source country.

σ_2 shows the importance of the parental time spent in the source country in shaping the level of human capital of the unskilled workers' child in time period $t+1$ while σ_1 shows the significance of the child's time spent on education in determining H_{t+1}. With regards to the role of parental absenteeism in the human capital formation of children, the possible interpretation of σ_2 is intriguing. The closer σ_2 is to zero, the less damaging parental absenteeism is on a child's accumulation of human capital. This parameter could capture a variety of social factors that could differ across households, communities, regions, or countries (such as marriage, presence of extended families, religious communities, and social networks in general). For instance, joint family systems, presence of extended families, cousin marriages, good social networks in terms of good neighbours are all likely to reduce the value of σ_2. A small value of σ_2 implies that a child's human capital formation is less sensitive to parental absenteeism, thereby reducing the negative impact of migration. On the other hand, nuclear families, broken marriages, absence of extended families and poor social networks are all likely to increase the value of σ_2, thereby making parental absenteeism due to migration more damaging for a child's accumulation of human capital.

The parents would allocate their income, I_t, to finance their own consumption, c_t, and the flow of remittances, θ_t, to their children. The total consumption, c_t, of the parents (unskilled adults) is the sum of consumption of the unskilled adult in the destination country, c_t^{UD}, and the consumption of the unskilled adult in the source country, c_t^{US} as shown:

$$c_t = c_t^{UD} + c_t^{US} \tag{2.1.3}$$

If the unskilled worker migrates to another country then he/she is able to earn sufficient income so as to finance his/her own consumption as well as the flow of remittances to his/her child dependent in the source country, i.e.:

$$I_t^{UD} = c_t^{UD} + \theta_t^{UD} \tag{2.1.4}$$

where I_t^{UD} is the income earned by the unskilled adult in the destination country and θ_t^{UD} is the remittance amount sent by the unskilled adult from the destination country to the child dependent in the source country. The income earned by the unskilled adult in the destination country is determined by the proportion of time the unskilled adult spends in the destination country and the unskilled worker's wage in the destination country as follows:

$$I_t^{UD} = \delta w_{UD} \tag{2.1.5}$$

Hence,

$$c_t^{UD} = I_t^{UD} - \theta_t^{UD}$$

$$c_t^{UD} = \delta w_{UD} - \theta_t^{UD} \tag{2.1.6}$$

Equation (2.1.6), hence, shows the consumption of the unskilled adult in the destination country. In the source country, the income earned by the unskilled adult, I_t^{US}, is determined by the proportion of time the unskilled adult spends in the source country and the unskilled worker's wage in the source country as follows:

$$I_t^{US} = (1 - \delta) w_{US} \tag{2.1.7}$$

However, in the source country, the unskilled worker is assumed to earn a subsistence income that can suffice only his/her own consumption and is unable to transfer any amount to the child dependent[2]. Hence, if the unskilled worker is working in the source country then he/she is not able to support his/her child's consumption, i.e.:

$$I_t^{US} = c_t^{US} \tag{2.1.8}$$

$$c_t^{US} = (1-\delta)w_{US} \tag{2.1.9}$$

In the source country, the children are not migrating so the consumption of a child, c_t^k, is satisfied by the budget constraint as follows:

$$c_t^k \leq \theta_t^{UD} + \theta_t^{US} + I_t^k \tag{2.1.10}$$

where I_t^k represents the income earned by the child by engaging in child labor and it is determined as shown:

$$I_t^k = (1-e_t)w_k \tag{2.1.11}$$

where $(1-e_t)$ denotes the time spent working by the child and w_k denotes the wage from child labor. By substituting in for (2.1.11) and since $\theta_t^{US} = 0$, the child's budget constraint appears to be as follows:

$$c_t^k \leq \theta_t^{UD} + (1-e_t)w_k \tag{2.1.12}$$

2.2 Production Sector

The source country contains both the skilled and the unskilled sectors of labor, producing the same identical good. The total output in the source country, Y_t, which is split up into the output of the skilled sector, $Y_t^{Skilled}$, and output of the unskilled sector, $Y_t^{Unskilled}$ is as follows:

$$Y_t = Y_t^{Skilled} + \varphi Y_t^{Unskilled}$$

[2]This is assumed to highlight the financial constraint that unskilled workers face in the source country.

where $\varphi \in (0,1)$. The output of the skilled sector, $Y_t^{Skilled}$, is determined by human capital of the skilled workers, H_t^S, as shown by the following linear production technology:

$$Y_t^S = H_t^S$$

Following Dessy & Rambeloma (2010), the unskilled sector of the source country is further divided into two sectors producing the same, identical good. One sector employs only the unskilled adults while the other sector employs children only[3]. The model is set up in an environment where child labor laws are not very effective. In order to produce the good, capital as well as land is required. Therefore, besides households, there also exist $\bar{K}$ childless capitalists[4]. Each capitalist has a one unit endowment of capital. Hence, $\bar{K}$ represents the total capitalists as well as the total capital stock of the economy. In this setting labor is hired by the capitalist and one unit of capital is required to begin a firm. A capitalist might begin a firm combining capital with adult labor to be used as inputs to production in the adult labor sector, or he might begin a firm combining capital with child labor in the child labor sector. Perfect mobility of capital is assumed across these two sectors. The total firms operative in the unskilled adult sector are denoted by K^U while K^k denotes those operative in child sector. Therefore, by normalization, the total capital stock in the sector $h \in \{U, k\}$ is K^h. The representative firm's output in adult sector is $Y_t^U = \left(L_t^U\right)^\mu$ while that in child sector is $Y_t^k = \phi\left(L_t^k\right)^\mu$, where Y_t^U denotes the output produced by the unskilled adults, Y_t^k denotes the output produced by the children, ϕ is the productivity parameter ($\phi \in (0,1)$), and μ denotes labor share (where $\mu \in (0,1)$). Hence, the total output of the unskilled sector, $Y_t^{Unskilled}$, is determined by the output produced by the unskilled adults, Y_t^U, and the output produced by the children, Y_t^k, in the following way:

$$Y_t^{Unskilled} = \rho Y_t^k + Y_t^U$$

[3] This is similar to think as if identical good is produced with two different technologies.

[4] This is done for simplicity purposes so as not to focus on the work/education decision for the children of capitalists.

where $\rho \in (0,1)$. Therefore, total output in the source country that is produced by both the skilled and unskilled sectors of labor is as follows:

$$Y_t = H_t^S + \varphi\left(\rho Y_t^k + Y_t^U\right)$$

In the unskilled sector of the source country, $L_t^U - M$ denotes the total supply of labor in adult sector while that in child sector is denoted by $\bar{L}_t^k$, where L_t^U is the total population of unskilled adults while M denotes the number of unskilled adults who migrate. $\bar{L}_t^k$ is also the economy-wide child labor incidence. The constraints regarding labor use as well as intersectoral allocation of capital respectively are:

$$(i)\ K^k L_t^k \leq \bar{L}_t^k \qquad (ii)\ K^U L_t^U \leq L_t^U - M \qquad (iii)\ K^U + K^k = \bar{K}$$

Following are the market-clearing wages under the scenario of perfect competition:

$$w_{US} = \mu\left(\frac{K^U}{L_t^U - M}\right)^{1-\mu} \tag{2.2.1}$$

$$w_k = \mu\phi\left(\frac{K^k}{\bar{L}_t^k}\right)^{1-\mu} \tag{2.2.2}$$

where w_{US} denotes the wage for the unskilled adult labor while w_k denotes the wage for child labor.

As the owner of the firm, a capitalist would claim a residual after production. This residual is $\pi^U = Y_t^U - w_{US}L_t^U$ for the adult sector and $\pi^k = Y_t^k - w_k L_t^k$ for the child sector. Thus, using (2.2.1) and (2.2.2) generates the returns to capital for the adult and child labor sectors respectively as follows:

$$\pi^U = \left(\frac{L_t^U - M}{\bar{K} - K^k}\right)^{\mu}(1-\mu) \tag{2.2.3}$$

$$\pi^k = \phi\left(\frac{\bar{L}_t^k}{K^k}\right)^{\mu}(1-\mu) \tag{2.2.4}$$

Child labor would not exist in the economy if $\pi^U > \pi^k$. Therefore, in this setting, a necessary condition for the emergence of child labor is that $\pi^U \leq \pi^k$. Since, capital is perfectly mobile across the sectors, it implies that the returns would be equalized across both the sectors in equilibrium, i.e. $\pi^U = \pi^k$. Plugging (2.2.3) and (2.2.4) into this condition yields the equilibrium capital allocation for both the sectors as follows:

$$K^k = \frac{\phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \qquad K^U = \frac{\left(L_t^U - M\right) \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \tag{2.2.5}$$

It can be clearly seen from (2.2.5) that when the unskilled workers migrate to the destination country, capital is reallocated to child sector from the unskilled adult sector, i.e., $\frac{dK^k}{dM} > 0$ while $\frac{dK^U}{dM} < 0$. Finally, by substituting (2.2.1) and (2.2.2) in (2.2.5) we arrive at the following wages for the unskilled adults and children respectively[5]:

$$w_{US} = \mu \left(\frac{\bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu} \tag{2.2.6}$$

$$w_k = \mu \phi^{\frac{1}{\mu}} \left(\frac{\bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu} \tag{2.2.7}$$

3 The Decision Problems of Parents

Parents jointly decide about the proportion of their time to be spent in the destination country (δ),the remittance amount, θ_t^{UD}, to be sent back to the child in the source country, as well as the proportion of time, e_t, their child would devote to education. Thus, the maximization problem of the household is as follows:

$$\max_{(\theta_t^{UD}, e_t, \delta)} \quad U_t = \ln\left(c_t\right) + \gamma \left[\ln c_t^k + \beta \ln H_{t+1}\right]$$

subject to:

$$c_t = c_t^{UD} + c_t^{US}$$

$$c_t^{UD} = I_t^{UD} - \theta_t^{UD}$$

$$c_t^{US} = I_t^{US}$$

[5] For detailed derivations: See Appendix A

$$I_t^{UD} = \delta w_{UD}$$

$$I_t^{US} = (1-\delta) w_{US}$$

$$c_t^k \leq \theta_t^{UD} + I_t^k$$

$$I_t^k = (1-e_t) w_k$$

$$H_{t+1} = \lambda H_t e_t^{\sigma_1} (1-\delta)^{\sigma_2}$$

$$\theta_t^{UD} \geq 0$$

$$0 \leq e_t \leq 1$$

$$0 \leq \delta \leq 1$$

All the budget constraints would be saturated in optimum. Therefore, the household value function is as follows:

$$V(\theta_t^{UD}, e_t, \delta) = \ln\left(\delta w_{UD} + (1-\delta) w_{US} - \theta_t^{UD}\right) + \gamma \left[\ln\left(\theta_t^{UD} + (1-e_t) w_k\right) + \beta \ln\left\{\lambda H_t e_t^{\sigma_1} (1-\delta)^{\sigma_2}\right\}\right] \tag{3.1}$$

The parents maximize this value function with respect to θ_t^{UD},e_t and δ. Therefore the maximization problem of the household can be reformulated as shown:

$$\max_{(\theta_t^{UD}, e_t, \delta)} V(\theta_t^{UD}, e_t, \delta)$$

In order to find the optimal amount of remittances to be sent back to the child in the source country, the optimal time devoted to education by the child, and the optimal proportion of total time of the unskilled adults to be spent in the destination country, following are the first order conditions for the optimisation problem with respect to θ_t^{UD}, e_t, and δ respectively:

$$\frac{dV}{d\theta_t^{UD}} = \frac{-1}{\delta w_{UD} + (1-\delta) w_{US} - \theta_t^{UD}} + \frac{\gamma}{\theta_t^{UD} + (1-e_t) w_k} = 0 \tag{3.2}$$

$$\frac{dV}{de_t} = \gamma \left(\frac{-w_k}{\theta_t^{UD} + (1-e_t) w_k} + \frac{\beta \sigma_1 \lambda H_t e_t^{\sigma_1 - 1} (1-\delta)^{\sigma_2}}{\lambda H_t e_t^{\sigma_1} (1-\delta)^{\sigma_2}} \right) = 0 \tag{3.3}$$

$$\frac{dV}{d\delta} = \frac{w_{UD} - w_{US}}{\delta w_{UD} + (1-\delta)w_{US} - \theta_t^{UD}} + \frac{\gamma\beta\sigma_2\lambda H_t e_t^{\sigma_1}(1-\delta)^{\sigma_2-1}(-1)}{\lambda H_t e_t^{\sigma_1}(1-\delta)^{\sigma_2}} = 0 \quad (3.4)$$

These first order conditions help us to arrive at the following three main equations connecting our three choice variables namely θ_t^{UD}, e_t, and δ:

$$\theta_t^{UD} = \frac{\gamma\delta(w_{UD} - w_{US}) + \gamma w_{US} - w_k + e_t w_k}{(1+\gamma)} \quad (3.5)$$

$$e_t = \frac{\beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k}{w_k(1+\sigma_1\beta)} \quad (3.6)$$

$$e_t = \frac{\beta\sigma_2\theta_t^{UD} + \beta\sigma_2 w_k + \delta(w_{UD} - w_{US}) - (w_{UD} - w_{US})}{\beta\sigma_2 w_k} \quad (3.7)$$

Equations (3.5), (3.6) and (3.7) exhibit the underlying relationship between θ_t^{UD}, e_t, and δ. According to these equations, if the unskilled adults spend a greater proportion of time in the destination country, it increases the remittance flows, thereby enhancing the children's time spent on education. Looking into the economic intuition behind this, there is a need to deeply analyse equations (3.5), (3.6) and (3.7). Equation (3.5) shows a positive relationship between δ and θ_t^{UD} implying that if the unskilled adults spend a greater proportion of their time in the destination country (i.e., a high δ) then this increases the amount that can be remitted back to the child dependent in the source country (i.e., θ_t^{UD} increases). Equation (3.6) reveals a positive relationship between θ_t^{UD} and e_t. This can be intuitively explained by revisiting the child's budget constraint $c_t^k \leq \theta_t^{UD} + (1-e_t)w_k$. The budget constraint shows two sources of financing child's consumption; remittances received from abroad and the income earned by engaging in child labor. The income earned by engaging in child labor supplements the remittances received for backing the child's consumption. When the emigrant parents remit back a larger amount of money (i.e., a high θ_t^{UD}), these remittances sent back by the migrants help to finance the consumption needs of the children, allowing them to postpone their participation in the market for child labor, hence extending their enrolment in school. Therefore, by shifting the use of children's time towards education and away from working, remittances play a major role to increase the time allocated to education (i.e., e_t increases) and hence, curb child labor. Moreover, equation (3.7) depicts a positive relationship between δ and e_t. This suggests that spending greater time in the destination country (i.e., a high δ) results in greater time devoted to education by the child (i.e.,

e_t increases) through the channel of increased remittances being sent back.

Solving the first order conditions simultaneously yields the optimal proportion of total time of the unskilled adults to be spent in the destination country (δ^*),the optimal amount of remittances, $\theta_t^{UD^*}$, to be sent back to the child in the source country, and the optimal proportion of time, e_t^*, that the child would devote to education respectively as follows[6]:

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]\,(w_{UD}-w_{US})}\right] \tag{3.8}$$

$$\theta_t^{UD^*} = \frac{\gamma(1+\sigma_1\beta)w_{UD} - w_k\,(1+\gamma\beta\sigma_2)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]} \tag{3.9}$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left(\frac{w_{UD}}{w_k}+1\right) \tag{3.10}$$

In order to ensure that all the optimal values are within the specified range, i.e. $\theta_t^{UD^*} \geq 0$, $0 \leq e_t^* \leq 1$ and $0 \leq \delta^* \leq 1$, the ratio of the wage earned by the unskilled worker in the source country to the wage earned by the children in the source country must lie in the following interval[7]:

$$\omega_A \simeq \frac{1}{\gamma(1+\sigma_1\beta)} < \frac{w_{US}}{w_k} < \frac{(1+\gamma)}{\gamma\beta\sigma_1} \simeq \omega_B \tag{3.11}$$

Looking at the corner solutions for e_t^* can have interesting implications. In the case where $e_t^* = 0$ would imply that the children of the unskilled workers would spend their entire time working, devoting no time to education. In such a scenario, children of the unskilled workers will accumulate a minimum level of human capital $\underline{H}$. $\underline{H}$ is very low and close to zero implying that these children would remain unskilled like their parents. On the contrary, if $e_t^* = 1$ then this implies that children of the unskilled workers are spending their entire time receiving education. In such a setup, there would be no child labor in the economy and children of the unskilled workers would accumulate higher levels of human capital.

[6] For proof: See Appendix B

[7] For proof: See Appendix C

4 Comparative Statics

In this section, we will be conducting a comparative static analysis of the optimal values δ^*, $\theta_t^{UD^*}$ and e_t^*. First we will analyze the important variables that affect the optimal proportion of total time of the unskilled adults to be spent in the destination country, δ^*, in (3.8).

Proposition 1 *The decision of the unskilled worker to migrate is determined by the following factors:*

(i) an increase in the unskilled worker's wage in the destination country (i.e., a high w_{UD}) encourages migration (i.e., δ^* increases);

(ii) an increase in the unskilled worker's wage in the source country (i.e., a high w_{US}) discourages migration (i.e., δ^* decreases);

(iii) an increase in the absolute difference between the unskilled worker's wage (i.e., a high $(w_{UD} - w_{US})$) encourages migration (i.e., δ^* increases);

(iv) an increase in the child's wage (i.e., a high w_k) discourages migration (i.e., δ^* decreases).

(v) a decrease in the sensitivity of H_{t+1} to the parental time spent in the source country (i.e., a low σ_2) encourages migration (i.e., δ^* increases).

Proposition 1 lists down several factors shaping the decision of the unskilled workers to migrate to the destination country. Looking into the economic intuition behind Proposition 1, it can be seen that whenever the wages offered to the unskilled workers abroad are going to increase, it incentivizes them to spend greater proportion of time in the destination country so as to benefit from the higher wages. Similarly, when the wages offered to the unskilled workers in the source country fall, it discourages them to stay in the source country and acts as an incentive for them to spend greater proportion of time in the destination country. Moreover, there exists not only the level effect of wages but the absolute difference between the wage that the unskilled worker earns abroad and the wage that the unskilled worker earns in the source country also exerts an impact on the decision of the unskilled worker to migrate. When the absolute difference between the unskilled worker's wage in the destination country and the source country grows, it motivates them to devote larger proportion of their time in the destination country to take advantage of the increasing wage spread. Besides, it is not just the unskilled worker's wage that affects δ^* but, interestingly, the child's wage also influences the unskilled worker's migration decision. When the children in the source country earn higher wages, it discourages the parents to spend more time abroad because one of the main reasons for migration is financing child's consumption. Higher child wage reduces the need to supplement the income earned by the child with remittances and encourages the parents to spend greater time in the country of origin.

Moreover, Proposition 1 brings to light an interesting connection between the the unskilled worker's migration decision and the sensitivity of child's human capital formation to the parental time spent in the source country. According to this proposition, if σ_2 decreases, then it encourages migration. A fall in σ_2 could occur in cases of joint family systems, presence of extended families, cousin marriages and good social networks in terms of good neighbours. In all such instances, there is a presence of someone to watch over the children left behind. This makes parental absenteeism less damaging to the child's accumulation of human capital. Thus, in such societies, the negative aspects of migration are reduced and the unskilled workers are motivated to spend greater proportion of time in the destination country and to benefit from the higher wages abroad.

Now we will be analyzing the key variables affecting the optimal amount of remittances, $\theta_t^{UD^*}$,and the optimal proportion of time, e_t^*, that the child would devote to education in equations (3.9) and (3.10) respectively.

Proposition 2 *An increase in the unskilled worker's wage in the destination country increases the amount of remittances which in turn increases the children's time spent on education. On the other hand, an increase in the child's wage reduces the remittance flows, consequently reducing the children's time spent on education.*

According to Proposition 2, the unskilled worker's wage in the destination country, w_{UD}, positively affects both the optimal amount of remittances, $\theta_t^{UD^*}$, and the optimal proportion of time, e_t^*, that the child would devote to education. On the other hand, the child's wage, w_k, has a negative relationship with both $\theta_t^{UD^*}$ and e_t^*. The intuitive explanation behind Proposition 2 is that when the wages offered to the unskilled workers in the destination country increase then they are left with a larger amount once their own consumption needs are met. Hence, the unskilled workers are able to send back larger remittances to their children in the source country. These larger amount of remittances sent back by the migrants help to finance the consumption needs of the children, shifting the use of children's time towards education and away from working. In this way, the child is able to devote greater time in acquiring education (i.e., e_t^* increases), hence, cutting down on the time spent working (i.e., $(1 - e_t^*)$ decreases) since both child labor as well as schooling hold competing claims as regards the time of a child. In this manner, increase in w_{UD} increases e_t^* through the channel of increased remittances.

Furthermore, it is not just the wage that the unskilled worker can earn abroad that determines the volume of remittances but the child's wage also influences the remittance flows. The child's wage, w_k, has a negative relationship with both $\theta_t^{UD^*}$ and e_t^*. The intuitive explanation behind this is that when the earning capacity of children increases, the education's opportunity cost rises. Due to this increase in the education's opportunity cost, investment in the human capital is discouraged. In such a scenario, even the parents who are altruistic would reduce the optimal amount of remittances, thereby shifting the use of children's time towards child labor and away from education. In this way, an increase in w_k reduces both $\theta_t^{UD^*}$ and e_t^*.

Interestingly, the wage that the unskilled worker earns in the source country, w_{US}, has no bearing on e_t^*. This is because the wage that is earned in the source country is just sufficient for the unskilled workers to fulfill their own consumption needs only. Thus, the parents are unable to finance their children's consumption, hence, not affecting the allocation of the children's time between working and education. For that reason, the international migration of unskilled workers is imperative as it has major implications for the human capital accumulation process of the children who are left behind.[8]

A detailed analysis of the optimal values δ^*, $\theta_t^{UD^*}$ and e_t^* can be undertaken if we incorporate the wages from the production sector. Using (2.2), (2.2.6) and (2.2.7), the optimal values in equations (3.8), (3.9) and (3.10) can be reformulated as follows[9]:

$$(1 - \delta^*) = \frac{\gamma\beta\sigma_2\left(\alpha + \phi^{\frac{1}{\mu}}\right)}{[1 + \gamma + \gamma\beta(\sigma_1 + \sigma_2)](\alpha - 1)} \tag{4.1}$$

$$\theta_t^{UD^*} = \frac{\mu\left(\bar{K}\right)^{1-\mu}}{[1 + \gamma + \gamma\beta(\sigma_1 + \sigma_2)]\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}}\left[\gamma(1 + \sigma_1\beta)\alpha - \phi^{\frac{1}{\mu}}(1 + \gamma\beta\sigma_2)\right] \tag{4.2}$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1 + \gamma + \gamma\beta(\sigma_1 + \sigma_2)]}\left(\frac{\alpha + \phi^{\frac{1}{\mu}}}{\phi^{\frac{1}{\mu}}}\right) \tag{4.3}$$

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[8]For proof: See Appendix D

[9]For detailed derivations: See Appendix E

where $(1 - \delta^*)$ is the optimal proportion of total time of the unskilled adults to be spent in the source country. Analysing equations (4.1), (4.2) and (4.3) reveals the key role of two main variables, namely α (i.e., the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country) and ϕ (i.e., the productivity of children) in the determination of our choice variables δ^*, $\theta_t^{UD^*}$ and e_t^*. A comparative static analysis brings to light a positive impact of α on δ^*, $\theta_t^{UD^*}$ and e_t^* while a negative impact on $(1 - \delta^*)$ [10]. Looking into its economic intuition, we can see that when the ratio of the wage offered to the unskilled worker in the destination country to the unskilled worker's wage in the source country increases, the unskilled workers view the rise in the relative wage as a positive stimulus. As a result, the unskilled workers choose to spend greater proportions of their time in the destination country while reducing their time spent in the home country so as to benefit from the much higher wages offered abroad. Resultantly, the unskilled emigrants are able to remit back a larger amount to their children. The children then use these larger remittances received to fulfill their consumption needs, therefore, cutting down the need to engage much in child labor. In this way, the children are able to increase the time devoted to education.

On the other hand, ϕ exerts a negative influence on δ^*, $\theta_t^{UD^*}$ and e_t^* while exerting a positive impact on $(1 - \delta^*)$. This means that an increase in the child's productivity (i.e., a high ϕ) reduces the optimal proportion of total time of the unskilled adults to be spent in the destination country, the optimal remittance flows and the optimal child's time spent on education while it increases the optimal proportion of total time of the unskilled adults to be spent in the source country. The economic intuition behind this is that when children become more productive (i.e., a high ϕ), the reward to children increases (i.e., the child wage,w_k, increases). Higher child wage reduces the need to supplement the income earned by the child with remittances. Therefore, an increase in the child's productivity reduces the optimal amount of remittances sent from abroad , hence, reducing the need for the unskilled workers to spend greater time in the destination country. For that reason, when the child becomes more productive, the unskilled adults choose to devote more time in the source country. Moreover, an increase in the child's productivity increases the reward for child labor. This acts as an incentive for the children to spend more time working so as to avail the increased incomes that can be earned. Resultantly, a child lowers down the time

[10] For proof: See Appendix F

allocated for education (i.e., an increase in ϕ increases w_k which in turn increases $(1 - e_t^*)$ and decreases e_t^*).

Where, on one hand, the international migration of unskilled adults increases the child's time spent on education, it also constructively lowers down child labor in the economy. The time spent working by the child, $(1 - e_t^*)$, and the total population of unskilled adults, L_t^U, determine the economy-wide child labor incidence, $\bar{L}_t^k$, as shown below:

$$\bar{L}_t^k = (1 - e_t^*)\, L_t^U \tag{4.4}$$

Using (4.3), we arrive at the economy-wide child labor incidence, $\bar{L}_t^k$, as follows[11]:

$$\bar{L}_t^k = \frac{\left[(1 + \gamma + \gamma\beta\sigma_2)\, \phi^{\frac{1}{\mu}} - \gamma\beta\sigma_1\alpha\right] L_t^U}{\left[1 + \gamma + \gamma\beta\left(\sigma_1 + \sigma_2\right)\right] \phi^{\frac{1}{\mu}}} \tag{4.5}$$

From (4.5) it can be seen that $\bar{L}_t^k$ is a positive function of ϕ (i.e., the productivity of children) and L_t^U (the total population of unskilled adults) and a negative function of α.(i.e., the unskilled worker's relative wage)[12].

Proposition 3 *An increase in the migration of unskilled workers and a rise in the unskilled worker's relative wage reduces the economy-wide child labor incidence while an increase in the productivity of children increases child labor in the economy.*

These results in Proposition 3 have an intuitive explanation. The reward for child labor increases with an increase in the productivity of the child which motivates the child to spend more time working, thus, increasing child labor. When the unskilled workers migrate to the destination country, the total population of unskilled adults falls. The increase in the number of unskilled migrants increases the remittance flows to the children in the source country. This reduces the economy-wide child labor incidence as the children then use these larger remittances received to cut down on the time spent working. Moreover, when the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country increases, the unskilled workers are tempted to migrate because of the higher wages being offered abroad. This, combined with the larger remittances that they can now afford to send back to

[11] For proof: See Appendix G

[12] For proof: See Appendix G

the child dependent, greatly helps the child to reduce the time spent working, thereby effectively bringing down child labor.

5 Human Capital Formation and Economic Growth

In this section, we shed light on the human capital formation process and the role of the international migration of unskilled labor in the economic growth of the source country. Using (4.1) and (4.3) we arrive at the level of human capital of the unskilled workers' child in time period $t+1$ as shown[13]:

$$H_{t+1} = \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \tag{5.1}$$

An in-depth analysis of (5.1) highlights the key role of two main variables in the child's human capital accumulation process, namely, the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country, α, and the child's productivity parameter, ϕ. The level of human capital of the unskilled workers' child in time period $t+1$ is a negative function of the child's productivity, ϕ. The intuitive explanation behind this is that as a child becomes more productive, the reward for child labor increases. This acts as a strong incentive for the child to switch to working, cutting down on the time that could have been productively spent on education. This is especially true in countries where both child labor as well as schooling hold competing claims as regards the time of a child. Resultantly, a rise in ϕ negatively affects the human capital accumulation of the child by reducing the optimal time devoted to education.

The most interesting influence on H_{t+1} is exerted by α. The unskilled worker's relative wage, α. affects H_{t+1} in a non-linear fashion. Looking into the intuitive explanation of this non-linear relationship is worthy of note. For this, let us revisit the human capital accumulation function of the child:

$$H_{t+1} = \lambda H_t \left(e_t^*\right)^{\sigma_1} \left(1-\delta^*\right)^{\sigma_2} \tag{5.2}$$

[13] For detailed derivations: See Appendix H

As already explained, a rise in the unskilled worker's relative wage causes the optimal proportion of time devoted to education by the child, e_t^*, to increase while the optimal proportion of total time of the unskilled adults to be spent in the source country, $(1-\delta^*)$, to decrease. This implies that a rise in α causes the unskilled parents to spend less time with their children. With an increase in the unskilled worker's relative wage, δ^* increases and consequently $(1-\delta^*)$ falls. Thus, the fall in $(1-\delta^*)$ is capturing the negative influence of parental absence because of migration. In a nutshell, whenever the unskilled worker's relative wage increases, the increase in e_t^* acts as a positive force on H_{t+1} while the fall in $(1-\delta^*)$ acts as a negative force on H_{t+1}. The overall impact of α on H_{t+1} depends on whether the positive force dominates the negative force or vice versa. The threshold value of α is as follows[14]:

$$\alpha = 1 + \frac{\sigma_2}{\sigma_1}\left(1 + \phi^{\frac{1}{\mu}}\right) = \alpha_A \tag{5.3}$$

Proposition 4 *When α is below the threshold level α_A, the emigration of unskilled workers exerts a negative influence on H_{t+1} (i.e., $\frac{dH_{t+1}}{d\alpha} < 0$ if $\alpha < \alpha_A$). On the other hand, when α is above the threshold level α_A, then the emigration of unskilled workers exerts a positive influence on H_{t+1} (i.e., $\frac{dH_{t+1}}{d\alpha} > 0$ if $\alpha > \alpha_A$).*

Simply, α shows the number of times the unskilled worker's wage in the destination country is greater than that in the source country. When α is below the threshold level α_A, then the emigration of unskilled workers exerts a negative influence on H_{t+1}. Having an α below α_A implies that the wage offered to the unskilled worker in the destination country is not high enough and so the unskilled worker sends back a lower volume of remittances. Due to this, the increase in the optimal proportion of time devoted to education by the child.is less. As a result, migration's positive effect of the increase in e_t^* is unable to overpower the negative effect of the fall in $(1-\delta^*)$ due to parental absenteeism, thereby generating an overall negative influence on H_{t+1}.

On the other hand, when α is above the threshold level α_A, then the emigration of unskilled workers exerts a positive influence on H_{t+1}. Having an α above α_A implies that the wage offered to the unskilled worker in the destination country is quite larger than that offered in

[14]For proof: See Appendix H

the source country and so the unskilled worker can afford to send back a substantial amount of remittances. The child then uses these larger remittances received to fulfill his/her consumption needs, therefore, cutting down the need to engage much in child labor. In this way, the child is able to greatly increase the time devoted to education. As a result, the positive force of the increase in e_t^* is able to overcome the negative effect of the fall in $(1 - \delta^*)$ due to parental absenteeism, thereby generating an overall positive influence on H_{t+1}. Thus, it can be clearly seen that the international migration of unskilled workers is beneficial for the human capital formation of children left behind only when the wages offered in the destination country are sufficiently larger so as to overwhelm the negative influence of the fall in the parent's time spent in the source country.

Equation (5.3) also gives us interesting insights about the relationship between σ_2 and α_A, whereby α_A is a positive function of σ_2. σ_2 shows the importance of the parental time spent in the source country in shaping the level of human capital of the unskilled workers' child in time period $t+1$. A larger value of σ_2 implies a larger value of α_A which in turn implies a larger range for the negative effect of the fall in $(1 - \delta^*)$ due to parental absenteeism on H_{t+1}. On the other hand, a smaller value of σ_2 implies a smaller value of α_A which in turn implies a smaller range for the negative effect of the fall in the parent's time spent in the source country on H_{t+1} and a larger range of the positive effect of the increase in e_t^* on H_{t+1}. Looking at the extreme value of σ_2 (i.e., if $\sigma_2 = 0$) gives us an even more interesting finding. If $\sigma_2 = 0$ then $\alpha_A = 1$. Since $w_{UD} > w_{US}$, so $\alpha > 1$.So if $\sigma_2 = 0$, then α would be above the threshold level α_A and we will only have a positive effect of international migration on child's human capital formation. Having $\sigma_2 = 0$ is similar to saying that only the child's time spent on education is important in determining H_{t+1} (i.e., only σ_1 matters) and in such instances we always see a positive impact of migration on child's human capital accumulation. Therefore, in cases of joint family systems, presence of extended families, cousin marriages and good social networks in terms of good neighbours, σ_2 is small and parental absenteeism is less damaging to the child's accumulation of human capital. This is because it reduces the negative effect of parental absenteeism on H_{t+1}.

Moreover, the relative importance of the parental time spent in the source country and the child's time spent on education (i.e., $\frac{\sigma_2}{\sigma_1}$) in the determination of the human capital of the future generation affects the size of the threshold value of α. σ_2 shows the importance of the parental time spent in the source country in shaping the level of human capital of the unskilled workers' child in time period $t+1$ while σ_1 shows the significance of the child's time spent on education in determining H_{t+1}. If the parental time spent in the source country is relatively more important than the child's time spent on education ($\sigma_2 > \sigma_1$) in determining the human capital of the future generation, then this this implies that the unskilled worker's relative wage has to be greater than a larger threshold value so as to produce an overall positive impact on H_{t+1}. This has an intuitive explanation. A rise in the unskilled worker's relative wage, α, causes the unskilled parents to spend less time with their children. The gender of the emigrant parent might affect the sensitivity of H_{t+1} to the fall in parents' time spent in the source country. If the human capital of the future generation is very sensitive as to whether the father emigrates or the mother emigrates, then this increases the value of σ_2, and consequently the ratio $\frac{\sigma_2}{\sigma_1}$. The increase in the ratio $\frac{\sigma_2}{\sigma_1}$ increases the size of the threshold value, α_A. As a result, the unskilled worker's relative wage has to be greater than a larger threshold value to overcome the fall in $(1-\delta^*)$ due to parental absenteeism so as to produce an overall positive impact on H_{t+1}.

On the other hand, if the parental time spent in the source country is relatively less important than the child's time spent on education in determining H_{t+1}, then this implies that the human capital of the future generation is less sensitive to parental absenteeism, reducing the size of σ_2, and consequently the ratio $\frac{\sigma_2}{\sigma_1}$. The lower value of $\frac{\sigma_2}{\sigma_1}$ reduces the size of the threshold value, α_A. As a result, the unskilled worker's relative wage has to be greater than a smaller threshold value to overpower the negative influence of parental absenteeism so as to produce an overall positive impact on H_{t+1}.

We can look at a special case where the parental time spent in the source country and the child's time spent on education are equally important (i.e., $\sigma_2 = \sigma_1$) in determining the human capital of the future generation. The threshold value of α in this case is as follows[15]:

$$\alpha = 2 + \phi^{\frac{1}{\mu}} = \alpha_B \tag{5.4}$$

When α is below the threshold level, α_B, i.e., if $\alpha < 2 + \phi^{\frac{1}{\mu}}$, then $\frac{dH_{t+1}}{d\alpha} < 0$. On the other hand, when α is above the threshold level, α_B, i.e., if $\alpha > 2 + \phi^{\frac{1}{\mu}}$, then $\frac{dH_{t+1}}{d\alpha} > 0$. This implies

[15] For proof: See Appendix H

that when the wages offered to the unskilled workers in the destination country are sufficiently larger (i.e., at least twice the wages that are being offered to the unskilled workers in the source country), then the migration of unskilled workers overwhelms the negative influence of parental absenteeism, thereby generating an overall positive influence on H_{t+1}. On the other hand, having an α below α_B implies that the increase in e_t^* is unable to overpower the negative effect of the fall in $(1-\delta^*)$ due to parental absenteeism, thereby generating an overall negative influence on H_{t+1}. In a nutshell, if the parental time spent in the source country and the child's time spent on education are equally important (i.e., $\sigma_2 = \sigma_1$) in determining the human capital of the future generation, then the international migration of unskilled workers is beneficial for the human capital formation of children left behind only when the unskilled workers can earn more than double the wages that they can earn while staying in the source country.

We will now see how the international migration of unskilled workers affects economic growth in the source country. For that we need to find the level of human capital of all the children in the economy in time period $t+1$ (i.e., H_{t+1}^E). H_{t+1}^E is a weighted average of the level of human capital of the skilled workers' child in time period $t+1$ (i.e., H_{t+1}^S) and the level of human capital of the unskilled workers' child in time period $t+1$ (i.e., H_{t+1}^U) as shown below:

$$H_{t+1}^E = H_{t+1}^S \left(\frac{L_t^S}{N_t}\right) + H_{t+1}^U \left(\frac{L_t^U}{N_t}\right) \tag{5.5}$$

where N_t is the total population in the source country, L_t^S is the total population of skilled adults and L_t^U is the total population of unskilled adults. Since we have been solving the problem of unskilled workers, the level of human capital of the unskilled workers' child in time period $t+1$ is:

$$H_{t+1}^U = \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \tag{5.6}$$

The children of the skilled population do not work. Hence, child labor in the skilled sector of the population is zero, i.e. $(1-e_t) = 0$. Therefore,children having skilled parents devote all their time to education, i.e. $e_t = 1$. Moreover, it is assumed that the skilled parents do not migrate and spend all their time in the source country. So in the skilled sector, $\delta = 0$ and $(1-\delta) = 1$.

Thus, the level of human capital of the skilled workers' child in time period $t+1$ is:

$$H_{t+1}^{S} = \lambda H_t \tag{5.7}$$

Using (2.1), (5.5), (5.6) and (5.7), we arrive at the following level of human capital of all the children in the economy in time period $t+1$:

$$H_{t+1}^{E} = \lambda H_t \left(\frac{L_t^S}{L_t^S + L_t^U} \right) + \left(\frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \right) \left(\frac{L_t^U}{L_t^S + L_t^U} \right) \tag{5.8}$$

From (5.8) it can be clearly seen[16] that H_{t+1}^{E} is a negative function of the child's productivity, ϕ. Moreover, as explained earlier, the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country, α. affects the level of human capital of the unskilled workers' child in time period $t+1$ (i.e., H_{t+1}^{U}) non-linearly, hence, affecting H_{t+1}^{E} in a non-linear manner. Furthermore, as already explained, the total population of unskilled adults, L_t^U, has a negative influence on the level of human capital of all the children in the economy in time period $t+1$. Therefore, with the international migration of unskilled workers, the total population of unskilled adults falls overtime, thereby enhancing the economy-wide level of human capital in time period $t+1$.

The growth of human capital can be calculated as follows:

$$g_h = \frac{H_{t+1}^{E}}{H_t} - 1 \tag{5.9}$$

where g_h is the rate at which the level of human capital of all the children in the economy in time period $t+1$ is growing. Using (5.8) and (5.9), we arrive at the following growth rate of human capital:

$$(1+g_h) = \lambda \left(\frac{L_t^S}{L_t^S + L_t^U} \right) + \left(\frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \right) \left(\frac{L_t^U}{L_t^S + L_t^U} \right) \tag{5.10}$$

[16] For proof: See Appendix I

Equation (5.10) depicts the growth rate of human capital, which is in effect the growth rate of output as well.[17] The unskilled sector of the source country cannot grow because $\frac{\dot{K}}{K} = 0$ in the steady state. Therefore, in our model, the primary element driving economic growth is human capital accumulation. Hence, output will grow at the rate at which human capital is growing.

Proposition 5 *The economy would grow faster when:*

(i) more unskilled adults migrate abroad (i.e., when L_t^U falls);

(ii) the productivity parameter in the human capital technology is high (i.e., λ is high);

(iii) children are less productive at work (i.e., ϕ is low); and

(iv) when the unskilled worker's relative wage is above the threshold level, α_A (i.e., $\alpha > \alpha_A$). However, the growth rate is adversely affected if $\alpha < \alpha_A$.

The economic intuition behind these results is that when the earnings of the unskilled workers abroad are sufficiently larger as compared to their wage in the source country, i.e., the unskilled worker's relative wage is above the threshold level, then they decide to spend greater time abroad to benefit from this wage differential. The higher wages abroad enable the unskilled workers to send back larger chunks of their income as remittances to the children in the country of origin. Children then effectively use these larger remittances received to bring down the time spent working and dedicate more time to acquire human capital. This overcomes the negative effects of parental absenteeism, which, in turn, exerts beneficial effects on the source country's growth rate. Moreover, the migration of unskilled workers implies that the total population of unskilled adults falls. The increasing number of unskilled migrants results in a larger remittance flow, thereby promoting economic growth in the source country. Furthermore, the economy would grow faster when the productivity parameter in the human capital technology is high (i.e., λ is high) which is also confirmed by Lucas (1988). When the country comprises of productive educational institutes, the efficiency of time invested in human capital increases and a child is able to accumulate higher levels of human capital which, in turn, promotes economic growth in the country. Besides, when the children are less productive at work, then the lower reward for labor demotivates them to engage in child labor. Alternately, children spend greater time building up their human capital, thereby, promoting economic growth in the source country.

However, when the unskilled worker's relative wage is below the threshold level α_A, then

[17] For proof: See Appendix I

the migration of unskilled workers adversely affects the growth rate. This is because parental absenteeism emerges as a dominant force in this case and hampers human capital formation of children left behind. This implies that caution must be exercised by countries using migration of unskilled workers as a tool to promote economic growth in the country of origin.

6 Conclusion

There exist a lot of studies that have empirically tested the implications of international migration for child education. However, this particular issue is rarely found in the theoretical field. This paper caters to this concern and provides a theoretical framework to formally illustrate the implications of the international migration patterns that we see today in the real world. This study explores the combined effect of parental absenteeism due to emigration of unskilled labor and remittances on the economic growth in the source country through the channel of human capital accumulation of children left behind. In this study, we design a proper theoretical model that clearly spells out the parameters and conditions under which the international migration of unskilled labor is beneficial for the economic growth of the source country. There is no existing unifying framework that integrates all the aspects, namely endogenous decision making of migration and child education, remittances, parental absenteeism and economic growth in a coherent framework.

The results of our study reveal that the decision of the unskilled workers to migrate is determined by the wages that unskilled workers face abroad and at home as well as the wage that their children earn in the source country. When the absolute difference between the unskilled worker's wage in the destination country and the source country grows, it motivates them to devote larger proportion of their time in the destination country to take advantage of the increasing wage spread. Besides, it is not just the unskilled worker's wage that affects their migration decision but, interestingly, the child's wage also influences unskilled migration. Higher child wage reduces the need to supplement the income earned by the child with remittances and encourages the parents to spend greater time in the country of origin, thereby discouraging migration.

The results of this study also highlight the role of the wage earned by unskilled workers in the destination country as well as the wage earned by their children in determining the amount of remittances. According to the results, an increase in the unskilled worker's wage in the destination country increases the amount of remittances which in turn increases the children's time spent on education by relaxing financial constraints. On the other hand, an increase in the child's wage reduces the remittance flows, consequently reducing the children's time spent on education.

The results of our study also bring to light the factors affecting child labor in the economy. According to the results, the international migration of unskilled adults and a rise in the unskilled worker's relative wage constructively lowers down the economy-wide child labor incidence while an increase in the productivity of children increases child labor in the economy. Moreover, the international migration of unskilled workers is beneficial for the human capital formation of children left behind and economic growth in the source country only when the wages offered in the destination country are sufficiently larger than the wages that are being offered to the unskilled workers in the source country. In such a scenario, the positive effect of unskilled migration, in the form of an increase in the optimal proportion of time devoted to education by the child, overpowers the negative effect of parental absenteeism. On the other hand, when the ratio of the unskilled worker's wage in the destination country to the unskilled worker's wage in the source country is below a threshold level then the migration of unskilled workers adversely affects the growth rate. This is because parental absenteeism emerges as a dominant force in this case and hampers human capital formation of children left behind.

Moreover, the size of the threshold value of the unskilled worker's relative wage is determined by the relative importance of the parental time spent in the source country and the child's time spent on education in the determination of the human capital of the future generation. If the human capital of the future generation is very sensitive to parental absenteeism then this increases the size of the threshold value. As a result, the unskilled worker's relative wage has to be greater than a larger threshold value to overcome parental absenteeism so as to produce an overall positive impact on children's human capital and economic growth. On the other hand, if the parental time spent in the source country is relatively less important than the child's time spent on education in determining the human capital of the future generation, then this implies that children's human capital is less sensitive to parental absenteeism, reducing the size of the threshold value. Consequently, the unskilled worker's relative wage has to be greater than a smaller threshold value to overpower the negative influence of parental absenteeism so as to exert an overall positive impact on children's human capital and economic growth. Finally, if the parental time spent in the source country and the child's time spent on education are equally important in determining the human capital of the future generation, then the international migration of unskilled workers is beneficial for the human capital formation of children left behind and economic growth only when the unskilled workers can earn more than double the wages that they can earn while staying in the source country.

The results of the study are applicable for children belonging to migrant households. Moreover, the results are dependent upon a threshold condition beyond which positive effects of international migration of unskilled labor are materialized. Furthermore, there might be a role of negative economic shocks in the real world which could result in a lower wage for unskilled workers in the source country. In such a situation, households experiencing migration might not be able to fill the gap despite the flow of remittances and children's time spent on education might not increase as predicted.

The results of the study bring to light important policy implications particularly for developing countries. The developing countries can encourage the migration of unskilled labor so as to curb child labor. The remittances sent back by the unskilled migrants play a major role to promote school enrolment and to reduce child labor.by shifting the use of children's time towards education and away from working. Moreover, countries can promote international migration of unskilled workers so as to foster economic growth in the source country. However, the results of our study imply that caution must be exercised by countries using migration of unskilled workers as a tool to promote economic growth in the country of origin. This infers that countries should encourage their unskilled workers to migrate to countries where they can earn sufficiently higher wages as compared to the wage in the home country. Similarly, if the unskilled workers are migrating from countries where wages are very low then unskilled migration would be quite beneficial as it increases the probability of raising the unskilled worker's relative wage above the threshold level that is required to boost economic growth in the country of origin.

Furthermore, as the results of the study have highlighted an important role of remittances in human capital formation of children and economic growth, they suggest that both the sending and receiving countries should tax remittances less which could in turn encourage remittances and prove to be beneficial for the development of poorer countries. Remittance flows are received directly by the poor families who are in need of it. For that reason, there is a dire need for developing countries to encourage remittances. In a nutshell, the international migration of unskilled labor can prove to be valuable for the source country given that the determining threshold conditions are met with effectively.

What this study has explored are possible ways in theory that remittances from foreign earned income can influence education choices, which in the constructed model affect human capital which in turn influences economic growth. Examining whether the effects at the macro level are marginal or substantial, especially trying to take into account parental absenteeism, may be a productive area for future research.

7 Appendix A

This appendix provides a detailed proof of how the wages for the unskilled adults and children are determined in the production sector. The representative firm's output in adult sector is as shown:

$$Y_t^U = \left(L_t^U\right)^{\mu}$$

Following is the unskilled adult labor use contraint:

$$K^U L_t^U \leq L_t^U - M$$

$$L_t^U \leq \frac{L_t^U - M}{K^U}$$

Derivating output produced by the unskilled adults with respect to unskilled adult labor yields the wage for the unskilled adult labor in the source country as follows:

$$w_{US} = \frac{dY_t^U}{dL_t^U}$$

$$w_{US} = \mu\left(L_t^U\right)^{\mu-1}$$

$$w_{US} = \mu\left(\frac{L_t^U - M}{K^U}\right)^{\mu-1}$$

$$w_{US} = \mu\left(\frac{K^U}{L_t^U - M}\right)^{1-\mu} \tag{1}$$

The representative firm's output in child sector is as shown:

$$Y_t^k = \phi\left(L_t^k\right)^{\mu}$$

Following is the child labor use contraint:

$$K^k L_t^k \leq \bar{L}_t^k$$

$$L_t^k = \frac{\bar{L}_t^k}{K^k}$$

Derivating the output produced by children with respect to child labor yields the wage for children as follows:

$$w_k = \frac{dY_t^k}{dL_t^k}$$

$$w_k = \mu\phi\left(L_t^k\right)^{\mu-1}$$

$$w_k = \mu\phi\left(\frac{\bar{L}_t^k}{K^k}\right)^{\mu-1}$$

$$w_k = \mu\phi\left(\frac{K^k}{\bar{L}_t^k}\right)^{1-\mu} \tag{2}$$

The residual claimed by the capitalist in adult sector is as shown:

$$\pi^U = Y_t^U - w_{US}L_t^U$$

$$\pi^U = \left(L_t^U\right)^{\mu} - w_{US}L_t^U$$

Since $w_{US} = \mu\left(L_t^U\right)^{\mu-1}$

$$\pi^U = \left(L_t^U\right)^{\mu} - \left(\mu\left(L_t^U\right)^{\mu-1}\right)L_t^U$$

$$\pi^U = \left(L_t^U\right)^{\mu} - \mu\left(L_t^U\right)^{\mu-1+1}$$

$$\pi^U = \left(L_t^U\right)^{\mu} - u\left(L_t^U\right)^{\mu}$$

$$\pi^U = \left(L_t^U\right)^{\mu}(1-\mu)$$

Since $L_t^U \leq \frac{L_t^U - M}{K^U}$:

$$\pi^U = \left(\frac{L_t^U - M}{K^U}\right)^{\mu}(1-\mu)$$

Since $K^U = \bar{K} - K^k$:

$$\pi^U = \left(\frac{L_t^U - M}{\bar{K} - K^k}\right)^{\mu}(1-\mu) \tag{3}$$

The residual claimed by the capitalist in child sector is as shown:

$$\pi^k = Y_t^k - w_k L_t^k$$

$$\pi^k = \phi \left(L_t^k\right)^\mu - w_k L_t^k$$

Since $w_k = \mu\phi \left(L_t^k\right)^{\mu-1}$:

$$\pi^k = \phi \left(L_t^k\right)^\mu - \left(\mu\phi \left(L_t^k\right)^{\mu-1}\right) L_t^k$$

$$\pi^k = \phi \left(L_t^k\right)^\mu - \mu\phi \left(L_t^k\right)^{\mu-1+1}$$

$$\pi^k = \phi \left(L_t^k\right)^\mu - \mu\phi \left(L_t^k\right)^\mu$$

$$\pi^k = \phi \left(L_t^k\right)^\mu (1-\mu)$$

Since $L_t^k = \frac{\bar{L}_t^k}{K^k}$:

$$\pi^k = \phi \left(\frac{\bar{L}_t^k}{K^k}\right)^\mu (1-\mu) \tag{4}$$

Equalization of the returns across the sectors implies that:

$$\pi^U = \pi^k$$

Plugging (3) and (4) in the above equation:

$$\left(\frac{L_t^U - M}{\bar{K} - K^k}\right)^\mu (1-\mu) = \phi \left(\frac{\bar{L}_t^k}{K^k}\right)^\mu (1-\mu)$$

$$\left(\frac{L_t^U - M}{\bar{K} - K^k}\right)^\mu = \phi \left(\frac{\bar{L}_t^k}{K^k}\right)^\mu$$

$$\left(\frac{L_t^U - M}{\bar{K} - K^k}\right)^{\mu * \frac{1}{\mu}} = \phi^{\frac{1}{\mu}} \left(\frac{\bar{L}_t^k}{K^k}\right)^{\mu * \frac{1}{\mu}}$$

$$\frac{L_t^U - M}{\bar{K} - K^k} = \phi^{\frac{1}{\mu}} \left(\frac{\bar{L}_t^k}{K^k}\right)$$

$$K^k \left(L_t^U - M\right) = \phi^{\frac{1}{\mu}} \bar{L}_t^k \left(\bar{K} - K^k\right)$$

$$K^k \left(L_t^U - M\right) = \phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K} - \phi^{\frac{1}{\mu}} \bar{L}_t^k K^k$$

$$K^k \left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k K^k = \phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}$$

$$K^k [\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k] = \phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}$$

$$K^k = \frac{\phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \tag{5}$$

where K^k shows the equilibrium capital allocation in child sector.

$$K^U = \bar{K} - K^k$$

$$K^U = \bar{K} - \frac{\phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k}$$

$$K^U = \frac{\left(L_t^U - M\right) \bar{K} + \phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K} - \phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k}$$

$$K^U = \frac{\left(L_t^U - M\right) \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \tag{6}$$

where K^U shows the equilibrium capital allocation in adult sector.

To find w_{US}:

$$w_{US} = \mu \left(\frac{K^U}{L_t^U - M} \right)^{1-\mu}$$

$$w_{US} = \frac{\mu}{(L_t^U - M)^{1-\mu}} (K^U)^{1-\mu}$$

Plug in K^U from (6) in the above equation:

$$w_{US} = \frac{\mu}{(L_t^U - M)^{1-\mu}} \left(\frac{\left(L_t^U - M\right) \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu}$$

$$w_{US} = \frac{\mu}{(L_t^U - M)^{1-\mu}} \left(\frac{\left(L_t^U - M\right)^{1-\mu} (\bar{K})^{1-\mu}}{\{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k\}^{1-\mu}} \right)$$

$$w_{US} = \mu \left(\frac{\bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu} \tag{7}$$

where w_{US} shows the final wage for unskilled adult labor in the source country.

To find w_k:

$$w_k = \mu\phi \left(\frac{K^k}{\bar{L}_t^k} \right)^{1-\mu}$$

$$w_k = \frac{\mu\phi}{\left(\bar{L}_t^k\right)^{1-\mu}} \left(K^k\right)^{1-\mu}$$

Plug in K^k from (5) in the above equation:

$$w_k = \frac{\mu\phi}{\left(\bar{L}_t^k\right)^{1-\mu}} \left(\frac{\phi^{\frac{1}{\mu}} \bar{L}_t^k \bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu}$$

$$w_k = \frac{\mu\phi}{\left(\bar{L}_t^k\right)^{1-\mu}} \left(\frac{\left(\phi^{\frac{1}{\mu}}\right)^{1-\mu} \left(\bar{L}_t^k\right)^{1-\mu} \left(\bar{K}\right)^{1-\mu}}{\{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k\}^{1-\mu}} \right)$$

$$w_k = \mu\phi^{1+\frac{1-\mu}{\mu}} \left(\frac{\bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu}$$

$$w_k = \mu\phi^{\frac{1}{\mu}} \left(\frac{\bar{K}}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}} \bar{L}_t^k} \right)^{1-\mu} \tag{8}$$

where w_k shows the final wage for the children in child sector.

8 Appendix B

This appendix provides a detailed proof of how the optimal values, δ^*, $\theta_t^{UD^*}$ and e_t^*, are derived. Following are the first order conditions of the maximisation problem:

$$\frac{dV}{d\theta_t^{UD}} = \frac{-1}{\delta w_{UD} + (1-\delta) w_{US} - \theta_t^{UD}} + \frac{\gamma}{\theta_t^{UD} + (1-e_t) w_k} = 0$$

$$\frac{1}{\delta w_{UD} + (1-\delta) w_{US} - \theta_t^{UD}} = \frac{\gamma}{\theta_t^{UD} + (1-e_t) w_k} \tag{1}$$

$$\frac{dV}{de_t} = \gamma \left(\frac{-w_k}{\theta_t^{UD} + (1-e_t) w_k} + \frac{\beta\sigma_1 \lambda H_t e_t^{\sigma_1 - 1} (1-\delta)^{\sigma_2}}{\lambda H_t e_t^{\sigma_1} (1-\delta)^{\sigma_2}} \right) = 0$$

$$\frac{w_k}{\theta_t^{UD} + (1-e_t) w_k} = \frac{\beta\sigma_1}{e_t} \tag{2}$$

$$\frac{dV}{d\delta} = \frac{w_{UD} - w_{US}}{\delta w_{UD} + (1-\delta) w_{US} - \theta_t^{UD}} + \frac{\gamma\beta\sigma_2 \lambda H_t e_t^{\sigma_1} (1-\delta)^{\sigma_2 - 1} (-1)}{\lambda H_t e_t^{\sigma_1} (1-\delta)^{\sigma_2}} = 0$$

$$\frac{w_{UD} - w_{US}}{\delta w_{UD} + (1-\delta) w_{US} - \theta_t^{UD}} = \frac{\gamma\beta\sigma_2}{(1-\delta)} \tag{3}$$

From equation (2):

$$e_t w_k = \beta\sigma_1\theta_t^{UD} + \beta\sigma_1(1-e_t)w_k$$

$$e_t w_k = \beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k - \beta\sigma_1 e_t w_k$$

$$e_t w_k + \beta\sigma_1 e_t w_k = \beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k$$

$$e_t(w_k + \beta\sigma_1 w_k) = \beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_c$$

$$e_t = \frac{\beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k}{w_k(1+\sigma_1\beta)} \tag{4}$$

Equation (4) shows e_t in terms of θ_t^{UD}.

From equation (3):

$$\frac{1}{\delta w_{UD} + (1-\delta)w_{US} - \theta_t^{UD}} = \frac{\gamma\beta\sigma_2}{(w_{UD}-w_{US})(1-\delta)} \tag{5}$$

Equating equations (1) and (5):

$$\frac{1}{\delta w_{UD} + (1-\delta)w_{US} - \theta_t^{UD}} = \frac{1}{\delta w_{UD} + (1-\delta)w_{US} - \theta_t^{UD}}$$

$$\frac{\gamma}{\theta_t^{UD} + (1-e_t)w_k} = \frac{\gamma\beta\sigma_2}{(w_{UD}-w_{US})(1-\delta)}$$

$$\beta\sigma_2\theta_t^{UD} + \beta\sigma_2(1-e_t)w_k = (w_{UD}-w_{US})(1-\delta)$$

$$\beta\sigma_2\theta_t^{UD} + \beta\sigma_2 w_k - \beta\sigma_2 e_t w_k = (w_{UD}-w_{US})(1-\delta)$$

$$\beta\sigma_2\theta_t^{UD} + \beta\sigma_2 w_k - (w_{UD}-w_{US})(1-\delta) = \beta\sigma_2 e_t w_k$$

$$e_t = \frac{\beta\sigma_2\theta_t^{UD} + \beta\sigma_2 w_k - (w_{UD}-w_{US})(1-\delta)}{\beta\sigma_2 w_k} \tag{6}$$

$$e_t = \frac{\beta\sigma_2\theta_t^{UD} + \beta\sigma_2 w_k + \delta(w_{UD}-w_{US}) - (w_{UD}-w_{US})}{\beta\sigma_2 w_k}$$

Equation (6) shows e_t in terms of θ_t^{UD} and δ.

Equating equations (4) and (6):

$$e_t = e_t$$

$$\frac{\beta\sigma_2\theta_t^{UD} + \beta\sigma_2 w_k - (w_{UD} - w_{US})(1-\delta)}{\beta\sigma_2 w_k} = \frac{\beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k}{w_k(1+\sigma_1\beta)}$$

$$\beta^2\sigma_2\sigma_1\theta_t^{UD} + \beta^2\sigma_2\sigma_1 w_k = \beta(1+\sigma_1\beta)\sigma_2\theta_t^{UD} + \beta(1+\sigma_1\beta)\sigma_2 w_k - (w_{UD} - w_{US})(1-\delta)(1+\sigma_1\beta)$$

$$\beta(1+\sigma_1\beta)\sigma_2\theta_t^{UD} + \beta(1+\sigma_1\beta)\sigma_2 w_k - (w_{UD} - w_{US})(1+\sigma_1\beta) + \delta(w_{UD} - w_{US})(1+\sigma_1\beta) = \beta^2\sigma_2\sigma_1\theta_t^{UD} + \beta^2\sigma_2\sigma_1 w_k$$

$$\delta(w_{UD} - w_{US})(1+\sigma_1\beta) = \beta^2\sigma_2\sigma_1\theta_t^{UD} + \beta^2\sigma_2\sigma_1 w_k - \beta(1+\sigma_1\beta)\sigma_2\theta_t^{UD} - \beta(1+\sigma_1\beta)\sigma_2 w_k + (w_{UD} - w_{US})(1+\sigma_1\beta)$$

$$\delta(w_{UD} - w_{US})(1+\sigma_1\beta) = \theta_t^{UD}\{\beta^2\sigma_2\sigma_1 - \beta(1+\sigma_1\beta)\sigma_2\} + w_k\{\beta^2\sigma_2\sigma_1 - \beta(1+\sigma_1\beta)\sigma_2\} + (w_{UD} - w_{US})(1+\sigma_1\beta)$$

$$\delta(w_{UD} - w_{US})(1+\sigma_1\beta) = \theta_t^{UD}(\beta^2\sigma_2\sigma_1 - \beta\sigma_2 - \beta^2\sigma_2\sigma_1) + w_k(\beta^2\sigma_2\sigma_1 - \beta\sigma_2 - \beta^2\sigma_2\sigma_1) + (w_{UD} - w_{US})(1+\sigma_1\beta)$$

$$\delta(w_{UD} - w_{US})(1+\sigma_1\beta) = -\beta\sigma_2\theta_t^{UD} - \beta\sigma_2 w_k + (w_{UD} - w_{US})(1+\sigma_1\beta)$$

$$\delta(w_{UD} - w_{US})(1+\sigma_1\beta) = (1+\sigma_1\beta)(w_{UD} - w_{US}) - \beta\sigma_2\theta_t^{UD} - \beta\sigma_2 w_k$$

$$\delta = \frac{(1+\sigma_1\beta)(w_{UD} - w_{US}) - \beta\sigma_2\theta_t^{UD} - \beta\sigma_2 w_k}{(1+\sigma_1\beta)(w_{UD} - w_{US})} \tag{7}$$

Equation (7) shows δ in terms of θ_t^{UD}.

From equation (1):

$$\theta_t^{UD} + (1-e_t)w_k = \gamma\delta w_{UD} + \gamma(1-\delta)w_{US} - \gamma\theta_t^{UD}$$

$$\theta_t^{UD} + \gamma\theta_t^{UD} = \gamma\delta w_{UD} + \gamma(1-\delta)w_{US} - (1-e_t)w_k$$

$$\theta_t^{UD}(1+\gamma) = \gamma\delta w_{UD} + \gamma w_{US} - \gamma\delta w_{US} - (1-e_t)w_k$$

$$\theta_t^{UD}(1+\gamma) = \gamma\delta w_{UD} - \gamma\delta w_{US} + \gamma w_{US} - w_k + e_t w_k$$

$$\theta_t^{UD}(1+\gamma) = \gamma\delta(w_{UD} - w_{US}) + \gamma w_{US} - w_k + e_t w_k$$

$$\theta_t^{UD} = \frac{\gamma\delta(w_{UD} - w_{US}) + \gamma w_{US} - w_k + e_t w_k}{(1+\gamma)} \tag{8}$$

Equation (8) shows θ_t^{UD} in terms of e_t and δ.

Putting Equations (4) and (7) in equation (8):

$$\theta_t^{UD}(1+\gamma) = \frac{\gamma(w_{UD} - w_{US})[(1+\sigma_1\beta)(w_{UD} - w_{US}) - \beta\sigma_2\theta_t^{UD} - \beta\sigma_2 w_k]}{(1+\sigma_1\beta)(w_{UD} - w_{US})} + \gamma w_{US} - w_k + w_k\left[\frac{\beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k}{w_k(1+\sigma_1\beta)}\right]$$

$$\theta_t^{UD}(1+\gamma)(1+\sigma_1\beta) = \gamma(1+\sigma_1\beta)(w_{UD}-w_{US}) - \gamma\beta\sigma_2\theta_t^{UD} - \gamma\beta\sigma_2 w_k + \gamma(1+\sigma_1\beta)w_{US} - w_k(1+\sigma_1\beta) + \beta\sigma_1\theta_t^{UD} + \beta\sigma_1 w_k$$

$$\theta_t^{UD}(1+\gamma)(1+\sigma_1\beta) + \gamma\beta\sigma_2\theta_t^{UD} - \beta\sigma_1\theta_t^{UD} = \gamma(1+\sigma_1\beta)(w_{UD}-w_{US}) - \gamma\beta\sigma_2 w_k + \gamma(1+\sigma_1\beta)w_{US} - w_k(1+\sigma_1\beta) + \beta\sigma_1 w_k$$

$$\begin{aligned}\theta_t^{UD}\left[(1+\gamma)(1+\sigma_1\beta) + \gamma\beta\sigma_2 - \sigma_1\beta\right] &= \{\gamma(1+\sigma_1\beta)w_{UD} - \gamma(1+\sigma_1\beta)w_{US} - \gamma\beta\sigma_2 w_k \\ &\quad + \gamma(1+\sigma_1\beta)w_{US} - w_k(1+\sigma_1\beta) + \beta\sigma_1 w_k\}\end{aligned}$$

$$\theta_t^{UD}\left(1+\sigma_1\beta+\gamma+\gamma\sigma_1\beta+\gamma\beta\sigma_2-\sigma_1\beta\right) = \gamma(1+\sigma_1\beta)w_{UD} - w_k\left[\gamma\beta\sigma_2 + (1+\sigma_1\beta) - \sigma_1\beta\right]$$

$$\theta_t^{UD}\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right] = \gamma(1+\sigma_1\beta)w_{UD} - w_k\left(\gamma\beta\sigma_2 + 1 + \sigma_1\beta - \sigma_1\beta\right)$$

$$\theta_t^{UD}\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right] = \gamma(1+\sigma_1\beta)w_{UD} - w_k\left(1+\gamma\beta\sigma_2\right)$$

$$\theta_t^{UD^*} = \frac{\gamma(1+\sigma_1\beta)w_{UD} - w_k\left(1+\gamma\beta\sigma_2\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]} \tag{9}$$

To find the optimal value of e_t i.e. e_t^*, use equation (4):

$$e_t^* = \frac{\beta\sigma_1\theta_t^{UD^*}}{w_k(1+\sigma_1\beta)} + \frac{\beta\sigma_1 w_k}{w_k(1+\sigma_1\beta)}$$

Plugging in the optimal value of θ_{UD} from equation (9):

$$e_t^* = \frac{\beta\sigma_1\gamma(1+\sigma_1\beta)w_{UD} - \beta\sigma_1 w_k\left(1+\gamma\beta\sigma_2\right)}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]} + \frac{\beta\sigma_1 w_k}{w_k(1+\sigma_1\beta)}$$

$$e_t^* = \frac{\beta\sigma_1\gamma(1+\sigma_1\beta)w_{UD} - \beta\sigma_1 w_k\left(1+\gamma\beta\sigma_2\right) + \beta\sigma_1 w_k\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\beta\sigma_1\gamma(1+\sigma_1\beta)w_{UD} - \beta\sigma_1 w_k(1+\gamma\beta\sigma_2 - 1 - \gamma - \gamma\beta\sigma_1 - \gamma\beta\sigma_2)}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\beta\sigma_1\gamma(1+\sigma_1\beta)w_{UD} - \beta\sigma_1 w_k(-\gamma\beta\sigma_1 - \gamma)}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\beta\sigma_1\gamma(1+\sigma_1\beta)w_{UD} - \beta\sigma_1 w_k(-\gamma)(\sigma_1\beta+1)}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\beta\gamma\sigma_1(1+\sigma_1\beta)w_{UD} + \gamma\beta\sigma_1 w_k(1+\sigma_1\beta)}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\beta\gamma\sigma_1(1+\sigma_1\beta)(w_{UD}+w_k)}{w_k(1+\sigma_1\beta)\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\gamma\beta\sigma_1(w_{UD}+w_k)}{w_k\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\left(\frac{w_{UD}}{w_k}+\frac{w_k}{w_k}\right)$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\left(\frac{w_{UD}}{w_k}+1\right) \tag{10}$$

To find the optimal value of δ i.e. δ^*, put equation (9) in equation (7):

$$\delta^* = \frac{(1+\sigma_1\beta)(w_{UD}-w_{US}) - \beta\sigma_2 w_k}{(1+\sigma_1\beta)(w_{UD}-w_{US})} - \frac{\beta\sigma_2\theta_t^{UD^*}}{(1+\sigma_1\beta)(w_{UD}-w_{US})}$$

$$\delta^* = \frac{(1+\sigma_1\beta)(w_{UD}-w_{US}) - \beta\sigma_2 w_k}{(1+\sigma_1\beta)(w_{UD}-w_{US})} - \left[\frac{\beta\sigma_2\gamma(1+\sigma_1\beta)w_{UD} - \beta\sigma_2 w_k(1+\gamma\beta\sigma_2)}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\right]$$

$$\delta^* = \frac{\begin{gathered}\{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right] - \beta\sigma_2 w_k\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right] \\ -\beta\sigma_2\gamma(1+\sigma_1\beta)w_{UD} + \beta\sigma_2 w_k(1+\gamma\beta\sigma_2)\}\end{gathered}}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$\delta^* = \frac{\begin{gathered}\{(1+\sigma_1\beta)w_{UD}\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right] - (1+\sigma_1\beta)w_{US}\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right] - \beta\sigma_2\gamma(1+\sigma_1\beta)w_{UD} \\ -\beta\sigma_2 w_k\left(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2-1-\gamma\beta\sigma_2\right)\}\end{gathered}}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$\delta^* = \frac{w_{UD}(1+\sigma_1\beta)\left(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2-\gamma\beta\sigma_2\right) - \left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right](1+\sigma_1\beta)w_{US} - \beta\sigma_2 w_k(\gamma+\gamma\beta\sigma_1)}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$\delta^* = \frac{w_{UD}(1+\sigma_1\beta)\left(1+\gamma+\gamma\beta\sigma_1\right) - (1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)(1+\sigma_1\beta)w_{US} - \beta\sigma_2\gamma(1+\sigma_1\beta)w_k}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$\delta^* = \frac{w_{UD}(1+\sigma_1\beta)\left(1+\gamma\beta\sigma_1+\gamma\right) - (1+\gamma+\gamma\beta\sigma_1)(1+\sigma_1\beta)w_{US} - \gamma\beta\sigma_2(1+\sigma_1\beta)w_{US} - \beta\gamma\sigma_2(1+\sigma_1\beta)w_k}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$\delta^* = \frac{(1+\sigma_1\beta)\left(1+\gamma+\gamma\beta\sigma_1\right)\left(w_{UD}-w_{US}\right) - \gamma\beta\sigma_2(1+\sigma_1\beta)w_{US} - \beta\gamma\sigma_2(1+\sigma_1\beta)w_k}{(1+\sigma_1\beta)(w_{UD}-w_{US})\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

$$\delta^* = \frac{(1+\sigma_1\beta)\,(1+\gamma+\gamma\beta\sigma_1)\,(w_{UD}-w_{US})}{(1+\sigma_1\beta)\,[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]\,(w_{UD}-w_{US})} - \left[\frac{\gamma\beta\sigma_2(1+\sigma_1\beta)w_{US}+\gamma\beta\sigma_2(1+\sigma_1\beta)w_k}{(1+\sigma_1\beta)\,[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]\,(w_{UD}-w_{US})}\right]$$

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]} - \left[\frac{\gamma\beta\sigma_2(1+\sigma_1\beta)(w_{US}+w_k)}{(1+\sigma_1\beta)\,[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]\,(w_{UD}-w_{US})}\right]$$

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]\,(w_{UD}-w_{US})}\right] \tag{11}$$

9 Appendix C

In this appendix, we provide a detailed proof of the optimal values so as to verify that all the optimal values are within the specified range, i.e. $\theta_t^{UD^*} \geq 0$, $0 \leq e_t^* \leq 1$ and $0 \leq \delta^* \leq 1$.

To ensure that $\theta_t^{UD^*} \geq 0$:

$$\theta_t^{UD^*} = \frac{\gamma(1+\sigma_1\beta)w_{UD} - w_k\,(1+\gamma\beta\sigma_2)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]} \geq 0$$

$$\gamma(1+\sigma_1\beta)w_{UD} - w_k\,(1+\gamma\beta\sigma_2) \geq 0$$

$$\gamma(1+\sigma_1\beta)w_{UD} \geq w_k\,(1+\gamma\beta\sigma_2)$$

$$\frac{w_{UD}}{w_k} \geq \frac{(1+\gamma\beta\sigma_2)}{\gamma(1+\sigma_1\beta)}$$

$$w_{UD} \geq \frac{(1+\gamma\beta\sigma_2)\,w_k}{\gamma(1+\sigma_1\beta)} \tag{1}$$

Ensuring that $e_t^* \geq 0$:

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left(\frac{w_{UD}}{w_k}+1\right) \geq 0$$

$$\frac{w_{UD}}{w_k}+1 \geq 0$$

$$\frac{w_{UD}+w_k}{w_k} \geq 0$$

$$w_{UD}+w_k \geq 0 \tag{2}$$

Ensuring that $e_t^* \leq 1$:

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]}\left(\frac{w_{UD}}{w_k}+1\right) \leq 1$$

$$\left(\frac{w_{UD}}{w_k}+1\right) \leq \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)}{\gamma\beta\sigma_1}$$

$$\frac{w_{UD}}{w_k} \leq \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)}{\gamma\beta\sigma_1} - 1$$

$$\frac{w_{UD}}{w_k} \leq \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2-\gamma\beta\sigma_1)}{\gamma\beta\sigma_1}$$

$$\frac{w_{UD}}{w_k} \leq \frac{1+\gamma\beta+\gamma}{\gamma\beta}$$

$$w_{UD} \leq \frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1} \tag{3}$$

Ensuring that $\delta^* \geq 0$:

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)}\right] \geq 0$$

$$\frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]} \geq \frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)}$$

$$\left(1+\gamma+\gamma\beta\sigma_1\right)\left(w_{UD}-w_{US}\right) \geq \gamma\beta\sigma_2(w_{US}+w_k)$$

$$\left(1+\gamma+\gamma\beta\sigma_1\right)w_{UD} \geq \gamma\beta\sigma_2 w_{US} + \gamma\beta\sigma_2 w_k + \left(1+\gamma+\gamma\beta\sigma_1\right)w_{US}$$

$$\left(1+\gamma+\gamma\beta\sigma_1\right)w_{UD} \geq (1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US} + \gamma\beta\sigma_2 w_k$$

$$w_{UD} \geq \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US} + \gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)} \tag{4}$$

Ensuring that $\delta^* \leq 1$:

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)}\right] \leq 1$$

$$\frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]} - 1 \leq \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)}\right]$$

$$\frac{1+\gamma+\gamma\beta\sigma_1-1-\gamma-\gamma\beta\sigma_1-\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]} \leq \frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)}$$

$$-\gamma\beta\sigma_2 \leq \frac{\gamma\beta\sigma_2(w_{US}+w_k)}{(w_{UD}-w_{US})}$$

$$\frac{-\gamma\beta\sigma_2}{\gamma\beta\sigma_2} \leq \frac{(w_{US}+w_k)}{(w_{UD}-w_{US})}$$

$$-1 \leq \frac{(w_{US}+w_k)}{(w_{UD}-w_{US})}$$

$$-w_{UD}+w_{US} \leq w_{US}+w_k$$

$$w_{US}+w_k \geq -w_{UD}+w_{US}$$

$$w_{US}+w_k+w_{UD}-w_{US} \geq 0$$

$$w_k+w_{UD} \geq 0 \tag{5}$$

So there are three restrictions:

$$1)\ w_{UD} \geq \frac{\left(1+\gamma\beta\sigma_2\right)w_k}{\gamma(1+\sigma_1\beta)}$$

$$2)\ w_{UD} \leq \frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1}$$

$$3)\ w_{UD} \geq \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)}$$

To check whether restriction 1) is greater or restriction 3):

$$w_{UD} \geq \frac{\left(1+\gamma\beta\sigma_2\right)w_k}{\gamma(1+\sigma_1\beta)}$$

Put the value of w_{UD} from restriction 3) in restriction 1):

$$\frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)} \gtreqless \frac{\left(1+\gamma\beta\right)w_k}{\gamma(1+\beta)}$$

$$\gamma(1+\sigma_1\beta)(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+(\gamma+\gamma\beta\sigma_1)\gamma\beta\sigma_2 w_k \gtreqless \left(1+\gamma+\gamma\beta\sigma_1\right)(1+\gamma\beta\sigma_2)w_k$$

$$\gamma(1+\sigma_1\beta)(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US} \gtreqless (1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2+\gamma^2\beta\sigma_2+\gamma^2\beta^2\sigma_1\sigma_2-\gamma^2\beta\sigma_2-\gamma^2\beta^2\sigma_1\sigma_2)w_k$$

$$\gamma(1+\sigma_1\beta)(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US} \gtreqless (1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_k$$

$$\gamma(1+\sigma_1\beta)w_{US} \gtreqless w_k$$

$$\gamma(1+\sigma_1\beta)w_{US} > w_k$$

$$\frac{w_{US}}{w_k} > \frac{1}{\gamma(1+\sigma_1\beta)} = \omega_A \tag{A}$$

If inequality A is satisfied then restriction 3) is greater than restriction 1). Hence, if restriction 3) is satisfied then restriction 1) is automatically satisfied. Therefore, only two conditions are left:

$$i)\ w_{UD} \geq \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)}$$

$$ii)\ w_{UD} \leq \frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1}$$

To check if $\frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1}$ is greater or less than $\frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)}$:

$$\frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1} \gtreqless \frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)}$$

$$(1+\gamma+\gamma\beta\sigma_2)(1+\gamma+\gamma\beta\sigma_1)\,w_k \gtreqless \gamma\beta\sigma_1(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma^2\beta^2\sigma_1\sigma_2 w_k$$

$$\left(1+\gamma+\gamma\beta\sigma_1+\gamma+\gamma^2+\gamma^2\beta\sigma_1+\gamma\beta\sigma_2+\gamma^2\beta\sigma_2+\gamma^2\beta^2\sigma_1\sigma_2\right)w_k \gtreqless \gamma\beta\sigma_1(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma^2\beta^2\sigma_1\sigma_2 w_k$$

$$\left(1+\gamma+\gamma\beta\sigma_1+\gamma+\gamma^2+\gamma^2\beta\sigma_1+\gamma\beta\sigma_2+\gamma^2\beta\sigma_2+\gamma^2\beta^2\sigma_1\sigma_2-\gamma^2\beta^2\sigma_1\sigma_2\right)w_k \gtreqless \gamma\beta\sigma_1(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}$$

$$(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_k+\gamma(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_k \gtreqless \gamma\beta\sigma_1(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}$$

$$(1+\gamma)(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_k \gtreqless \gamma\beta\sigma_1(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}$$

$$(1+\gamma)w_k \gtreqless \gamma\beta\sigma_1 w_{US}$$

$$\frac{w_k}{w_{US}} > \frac{\gamma\beta\sigma_1}{1+\gamma}$$

$$\frac{w_{US}}{w_k} < \frac{(1+\gamma)}{\gamma\beta\sigma_e} = \omega_B \tag{B}$$

If inequality B is satisfied then $\frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1}$ is greater than $\frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)}$. Hence,

$$\frac{(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)w_{US}+\gamma\beta\sigma_2 w_k}{(1+\gamma+\gamma\beta\sigma_1)} \leq w_{UD} \leq \frac{(1+\gamma+\gamma\beta\sigma_2)w_k}{\gamma\beta\sigma_1}$$

Comparing whether ω_A is greater or less than ω_B:

$$\omega_A \gtreqless \omega_B$$

$$\frac{1}{\gamma(1+\sigma_1\beta)} \gtreqless \frac{(1+\gamma)}{\gamma\beta\sigma_1}$$

$<$

$$\beta\sigma_1 \gtreqless (1+\gamma)(1+\sigma_1\beta)$$

$$\beta\sigma_1 \gtreqless 1+\sigma_1\beta+\gamma+\gamma\sigma_1\beta$$

$$0 < 1+\sigma_1\beta+\gamma+\gamma\sigma_1\beta-\beta\sigma_1$$

$$0 < 1+\gamma\,(1+\sigma_1\beta)$$

$$1+\gamma(1+\sigma_1\beta) > 0$$

Therefore, $\omega_B > \omega_A$. Hence, ω_A shows the lower limit of the ratio of the wage earned by the unskilled worker in the source country to the wage earned by the children in the source country while ω_B shows the upper limit of the ratio of the wage earned by the unskilled worker in the source country to the wage earned by the children in the source country, i.e.:

$$\omega_A < \omega_{Uk} < \omega_B$$

where $\omega_{Uk} = \frac{w_{US}}{w_k}$ is the ratio of the wage earned by the unskilled worker in the source country to the wage earned by the children in the source country. Thus, ω_{Uk} lies in the following interval:

$$\omega_A \simeq \frac{1}{\gamma(1+\sigma_1\beta)} < \frac{w_{US}}{w_k} < \frac{(1+\gamma)}{\gamma\beta\sigma_1} \simeq \omega_B \tag{C}$$

The inequality C ensures that all the optimal values are within the specified range, i.e. $\theta_t^{UD^*} \geq 0$,

$0 \leq e_t^* \leq 1$ and $0 \leq \delta^* \leq 1$.

10 Appendix D

In this appendix, we will be presenting a detailed comparative static analysis of the optimal values δ^*, $\theta_t^{UD^*}$ and e_t^*. Following is the optimal proportion of total time of the unskilled adults to be spent in the destination country:

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})}\right] \tag{1}$$

Derivating δ^* with respect to the child wage, w_k shows a negative relationship:

$$\frac{d\delta^*}{dw_k} = -\frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})}$$

$$\frac{d\delta^*}{dw_k} < 0$$

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)(w_{UD}-w_{US})^{-1}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\right]$$

Derivating δ^* with respect to w_{UD} shows a positive impact of the unskilled worker's wage earned in the destination country on δ^*:

$$\frac{d\delta^*}{dw_{UD}} = \frac{-(-1)\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})^2}$$

$$\frac{d\delta^*}{dw_{UD}} = \frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})^2}$$

$$\frac{d\delta^*}{dw_{UD}} > 0$$

Derivating δ^* with respect to w_{US} shows a negative impact of the unskilled worker's wage earned in the source country on δ^*:

$$\frac{d\delta^*}{dw_{US}} = -\frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left[\frac{(w_{UD}-w_{US})(1)-(w_{US}+w_k)(-1)}{(w_{UD}-w_{US})^2}\right]$$

$$\frac{d\delta^*}{dw_{US}} = -\frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left[\frac{(w_{UD}-w_{US})+w_{US}+w_k}{(w_{UD}-w_{US})^2}\right]$$

$$\frac{d\delta^*}{dw_{US}} = -\frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left[\frac{w_{UD}+w_k}{(w_{UD}-w_{US})^2}\right]$$

$$\frac{d\delta^*}{dw_{US}} < 0$$

Derivating δ^* with respect to $(w_{UD}-w_{US})$ shows a positive impact of the absolute wage differential on δ^*:

$$\frac{d\delta^*}{d(w_{UD}-w_{US})} = -\frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left[\frac{(w_{US}+w_k)(-1)}{(w_{UD}-w_{US})^2}\right]$$

$$\frac{d\delta^*}{d(w_{UD}-w_{US})} = \frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})^2}$$

$$\frac{d\delta^*}{d(w_{UD}-w_{US})} > 0$$

Derivating δ^* with respect to σ_2 shows a negative impact of the sensitivity of H_{t+1} to parental time spent in the source country on δ^*:

$$\begin{aligned}\frac{d\delta^*}{d\sigma_2} &= \frac{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](0)-(1+\gamma+\gamma\beta\sigma_1)\gamma\beta}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^2} \\ &\quad -\frac{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\gamma\beta(w_{US}+w_k)-\gamma\beta\sigma_2(w_{US}+w_k)\gamma\beta(w_{UD}-w_{US})\}}{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\}^2}\end{aligned}$$

$$\begin{aligned}\frac{d\delta^*}{d\sigma_2} &= -\frac{(1+\gamma+\gamma\beta\sigma_1)\gamma\beta}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^2} \\ &\quad -\frac{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\gamma\beta(w_{US}+w_k)-\gamma\beta\sigma_2(w_{US}+w_k)\gamma\beta(w_{UD}-w_{US})\}}{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\}^2}\end{aligned}$$

$$\begin{aligned}\frac{d\delta^*}{d\sigma_2} &= -\frac{(1+\gamma+\gamma\beta\sigma_1)\gamma\beta(w_{UD}-w_{US})^2}{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^2(w_{UD}-w_{US})\}^2} \qquad \text{(i)} \\ &\quad -\frac{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\gamma\beta(w_{US}+w_k)-\gamma\beta\sigma_2(w_{US}+w_k)\gamma\beta(w_{UD}-w_{US})\}}{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\}^2}\end{aligned}$$

The first term in the above expression is A i.e. A $= -\frac{(1+\gamma+\gamma\beta\sigma_1)\gamma\beta(w_{UD}-w_{US})^2}{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^2(w_{UD}-w_{US})\}^2}$ while the second term is B i.e. B $= -\frac{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\gamma\beta(w_{US}+w_k)-\gamma\beta\sigma_2(w_{US}+w_k)\gamma\beta(w_{UD}-w_{US})\}}{\{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})\}^2}$.

We will now simplify the numerator of the term B as follows:

$$(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2)\left(w_{UD}-w_{US}\right)\gamma\beta\left(w_{US}+w_k\right)-\gamma\beta\sigma_2\left(w_{US}+w_k\right)\gamma\beta(w_{UD}-w_{US})$$

$$(1+\gamma+\gamma\beta\sigma_1)\left(w_{UD}-w_{US}\right)\gamma\beta\left(w_{US}+w_k\right)+\gamma\beta\sigma_2(w_{UD}-w_{US})\gamma\beta\left(w_{US}+w_k\right)-\gamma\beta\sigma_2\left(w_{US}+w_k\right)\gamma\beta(w_{UD}-w_{US})$$

$$(1+\gamma+\gamma\beta\sigma_1)\left(w_{UD}-w_{US}\right)\gamma\beta\left(w_{US}+w_k\right) \tag{ii}$$

Now we can put equation (ii) in equation (i):

$$\frac{d\delta^*}{d\sigma_2}=-\frac{(1+\gamma+\gamma\beta\sigma_1)\,\gamma\beta(w_{UD}-w_{US})^2}{\{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]^2\left(w_{UD}-w_{US}\right)\}^2}-\frac{(1+\gamma+\gamma\beta\sigma_1)\left(w_{UD}-w_{US}\right)\gamma\beta\left(w_{US}+w_k\right)}{\{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)\}^2}$$

$$\frac{d\delta^*}{d\sigma_2}=-\frac{[\gamma\beta\left(1+\gamma+\gamma\beta\sigma_1\right)\left(w_{UD}-w_{US}\right)]\left[\left(w_{UD}-w_{US}\right)+\left(w_{US}+w_k\right)\right]}{\{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)\}^2}$$

$$\frac{d\delta^*}{d\sigma_2}=-\frac{[\gamma\beta\left(1+\gamma+\gamma\beta\sigma_1\right)\left(w_{UD}-w_{US}\right)]\left[w_{UD}+w_k\right]}{\{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]\left(w_{UD}-w_{US}\right)\}^2} \tag{iii}$$

For equation (iii), the denominator is positive and the numerator is negative as long as $w_{UD} > w_{US}$. According to equation (2.2), $w_{UD} = \alpha w_{US}$ and $\alpha > 1$ so $w_{UD} > w_{US}$. Hence the numerator of equation (iii) is negative. Therefore:

$$\frac{d\delta^*}{d\sigma_2} < 0$$

Following is the optimal amount of remittances to be sent back:

$$\theta_t^{UD^*}=\frac{\gamma(1+\sigma_1\beta)w_{UD}-w_k\left(1+\gamma\beta\sigma_2\right)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]} \tag{2}$$

Derivating $\theta_t^{UD^*}$ with respect to w_{UD} reveals a positive impact of the unskilled worker's wage earned in the destination country on $\theta_t^{UD^*}$:

$$\frac{d\theta_t^{UD^*}}{dw_{UD}}=\frac{\gamma(1+\sigma_1\beta)}{[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)]}$$

$$\frac{d\theta_t^{UD^*}}{dw_{UD}} > 0$$

Derivating $\theta_t^{UD^*}$ with respect to w_k reveals a negative impact of the child's wage on $\theta_t^{UD^*}$:

$$\frac{d\theta_t^{UD^*}}{dw_k} = -\frac{(1+\gamma\beta\sigma_2)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}$$

$$\frac{d\theta_t^{UD^*}}{dw_k} < 0$$

Equation (3) shows the optimal time devoted to education by the child:

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{w_{UD}}{w_k}+1\right) \tag{3}$$

Derivating e_t^* with respect to w_{UD} depicts a positive effect of the unskilled worker's wage earned in the destination country on e_t^*:

$$\frac{de_t^*}{dw_{UD}} = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]\,w_k}$$

$$\frac{de_t^*}{dw_{UD}} > 0$$

Derivating e_t^* with respect to w_k shows a negative impact of child wage on e_t^*:

$$\frac{de_t^*}{dw_c} = -\frac{\gamma\beta\sigma_1 w_{UD}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]\,(w_k)^2}$$

$$\frac{de_t^*}{dw_k} < 0$$

Derivating e_t^* with respect to w_{US} reveals that there is no relationship between the unskilled worker's wage earned in the source country and e_t^*:

$$\frac{de_t^*}{dw_{US}} = 0$$

11 Appendix E

In this appendix, we provide a detailed proof of incorporating the wages from the production sector into the optimal values of δ^*, $\theta_t^{UD^*}$ and e_t^*. Following is the optimal time devoted to education by the child:

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left(\frac{w_{UD}}{w_k}+1\right)$$

Since $w_{UD} = \alpha w_{US}$:

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left(\frac{\alpha w_{US}}{w_k}+1\right)$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left[\frac{\alpha\mu\left(\frac{\bar{K}}{(L_t^U-M)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}}{\mu\phi^{\frac{1}{\mu}}\left(\frac{\bar{K}}{(L_t^U-M)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}}+1\right]$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left(\frac{\alpha}{\phi^{\frac{1}{\mu}}}+1\right)$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left(\frac{\alpha+\phi^{\frac{1}{\mu}}}{\phi^{\frac{1}{\mu}}}\right) \tag{1}$$

Equation (1) shows the optimal time devoted to education by the child when wages from the production sector are also incorporated. Following is the optimal amount of remittances to be sent back to the child in the source country:

$$\theta_t^{UD^*} = \frac{\gamma(1+\sigma_1\beta)w_{UD} - w_k\,(1+\gamma\beta\sigma_2)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}$$

Since $w_{UD} = \alpha w_{US}$:

$$\theta_t^{UD^*} = \frac{\gamma(1+\sigma_1\beta)\alpha w_{US} - w_k\,(1+\gamma\beta\sigma_2)}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}$$

$$\theta_t^{UD^*} = \frac{1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left[\begin{array}{c}\gamma(1+\sigma_1\beta)\alpha\mu\left(\frac{\bar{K}}{(L_t^U-M)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}\\ -(1+\gamma\beta\sigma_2)\mu\phi^{\frac{1}{\mu}}\left(\frac{\bar{K}}{(L_t^U-M)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}\end{array}\right]$$

$$\theta_t^{UD^*} = \frac{1}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]}\left[\left(\frac{\bar{K}}{\left(L_t^U-M\right)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}\mu\{\gamma(1+\sigma_1\beta)\alpha-\phi^{\frac{1}{\mu}}\,(1+\gamma\beta\sigma_2)\}\right]$$

$$\theta_t^{UD^*} = \frac{\mu\left(\bar{K}\right)^{1-\mu}}{[1+\gamma+\gamma\beta\,(\sigma_1+\sigma_2)]\left(L_t^U-M\right)+\phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}}\left[\gamma(1+\sigma_1\beta)\alpha-\phi^{\frac{1}{\mu}}\,(1+\gamma\beta\sigma_2)\right] \tag{2}$$

Equation (2) shows the optimal amount of remittances to be sent back to the child in the source country with the inclusion of wages derived from the production sector. Following is the optimal proportion of total time of the unskilled adults to be spent in the destination country:

$$\delta^* = \frac{(1+\gamma+\gamma\beta\sigma_1)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]} - \left[\frac{\gamma\beta\sigma_2(w_{US}+w_k)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})}\right]$$

$$\delta^* = \frac{(w_{UD}-w_{US})(1+\gamma+\gamma\beta\sigma_1) - \gamma\beta\sigma_2(w_{US}+w_k)}{(w_{UD}-w_{US})[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}$$

To find $(1-\delta^*)$:

$$(1-\delta^*) = 1 - \left[\frac{(w_{UD}-w_{US})(1+\gamma+\gamma\beta\sigma_1) - \gamma\beta\sigma_2(w_{US}+w_k)}{(w_{UD}-w_{US})[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\right]$$

$$(1-\delta^*) = \frac{(w_{UD}-w_{US})(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2) - (w_{UD}-w_{US})(1+\gamma+\gamma\beta\sigma_1) + \gamma\beta\sigma_2(w_{US}+w_k)}{(w_{UD}-w_{US})[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}$$

$$(1-\delta^*) = \frac{(w_{UD}-w_{US})(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2-1-\gamma\beta\sigma_1-\gamma) + \gamma\beta\sigma_1(w_{US}+w_k)}{(w_{UD}-w_{US})[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}$$

$$(1-\delta^*) = \frac{(w_{UD}-w_{US})\gamma\beta\sigma_2 + \gamma\beta\sigma_2(w_{US}+w_k)}{(w_{UD}-w_{US})[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2 w_{UD} - \gamma\beta\sigma_2 w_{US} + \gamma\beta\sigma_2 w_{US} + \gamma\beta\sigma_2 w_k}{(w_{UD}-w_{US})[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2(w_{UD}+w_k)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](w_{UD}-w_{US})}$$

Since $w_{UD} = \alpha w_{US}$:

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{\alpha w_{US}+w_k}{\alpha w_{US}-w_{US}}\right)$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{\alpha w_{US}+w_k}{w_{US}(\alpha-1)}\right)$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{\alpha w_{US}}{w_{US}(\alpha-1)} + \frac{w_k}{w_{US}(\alpha-1)}\right)$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{\alpha}{\alpha-1} + \frac{w_k}{w_{US}(\alpha-1)}\right)$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](\alpha-1)}\left(\alpha+\frac{w_k}{w_{US}}\right)$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](\alpha-1)}\left[\alpha+\frac{\mu\phi^{\frac{1}{\mu}}\left(\frac{\bar{K}}{(L_t^U-M)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}}{\mu\left(\frac{\bar{K}}{(L_t^U-M)+\phi^{\frac{1}{\mu}}\bar{L}_t^k}\right)^{1-\mu}}\right]$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](\alpha-1)}\left(\alpha+\phi^{\frac{1}{\mu}}\right)$$

$$(1-\delta^*) = \frac{\gamma\beta\sigma_2\left(\alpha+\phi^{\frac{1}{\mu}}\right)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](\alpha-1)} \tag{3}$$

Equation (3) shows the optimal proportion of total time of the unskilled adults to be spent in the destination country when the wages from the production sector have been included.

12 Appendix F

In this appendix, we conduct a detailed comparative static analysis of the optimal values of δ^*, $\theta_t^{UD^*}$ and e_t^* which have incorporated the wages from the production sector. Following is the optimal time devoted to education by the child:

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{\alpha}{\phi^{\frac{1}{\mu}}}+1\right)$$

Derivating the optimal time devoted to education by the child with respect to α shows a positive impact of α on e_t^*:

$$\frac{de_t^*}{d\alpha} = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]\,\phi^{\frac{1}{\mu}}}$$

$$\frac{de_t^*}{d\alpha} > 0$$

$$e_t^* = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\alpha\phi^{-\frac{1}{\mu}}+1\right)$$

Derivating the optimal time devoted to education by the child with respect to ϕ shows a negative impact of ϕ on e_t^*:

$$\frac{de_t^*}{d\phi} = \frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(-\frac{1}{\mu}\alpha\phi^{-\frac{1}{\mu}-1}\right)$$

$$\frac{de_t^*}{d\phi} = -\frac{\gamma\beta\sigma_1}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left(\frac{\alpha}{\phi^{\frac{1+\mu}{\mu}}}\right)$$

$$\frac{de_t^*}{d\phi} = -\frac{\gamma\beta\sigma_1\alpha}{\mu[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]\left(\phi^{\frac{1+\mu}{\mu}}\right)}$$

$$\frac{de_t^*}{d\phi} < 0$$

Following is the optimal amount of remittances to be sent back to the child in the source country:

$$\theta_t^{UD^*} = \frac{\mu\left(\bar{K}\right)^{1-\mu}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}}\left[\gamma(1+\sigma_1\beta)\alpha - \phi^{\frac{1}{\mu}}(1+\gamma\beta\sigma_2)\right]$$

Derivating the optimal amount of remittances to be sent back to the child in the source country with respect to α depicts a positive relationship between α and $\theta_t^{UD^*}$:

$$\frac{d\theta_t^{UD^*}}{d\alpha} = \frac{\mu\left(\bar{K}\right)^{1-\mu}\gamma(1+\sigma_1\beta)}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)](L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}}$$

$$\frac{d\theta_t^{UD^*}}{d\alpha} > 0$$

$$\theta_t^{UD^*} = \frac{\mu\left(\bar{K}\right)^{1-\mu}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left[\frac{\gamma(1+\sigma_1\beta)\alpha - \phi^{\frac{1}{\mu}}(1+\gamma\beta\sigma_2)}{\left(L_t^U - M\right) + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}}\right]$$

Derivating the optimal amount of remittances to be sent back to the child in the source country with respect to ϕ depicts a negative relationship between ϕ and $\theta_t^{UD^*}$:

$$\frac{d\theta_t^{UD^*}}{d\phi} = \frac{\mu\left(\bar{K}\right)^{1-\mu}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]}\left[\frac{\begin{array}{c}(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}\{-\frac{1}{\mu}\phi^{\frac{1}{\mu}-1}(1+\gamma\beta\sigma_2)\}- \\ \{\gamma(1+\sigma_1\beta)\alpha - \phi^{\frac{1}{\mu}}(1+\gamma\beta\sigma_2)\} \\ \{(1-\mu)(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{-\mu}\left(\frac{1}{\mu}\phi^{\frac{1}{\mu}-1}\bar{L}_t^k\right)\}\end{array}}{(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{2-2\mu}}\right]$$

$$\frac{d\theta_t^{UD^*}}{d\phi} = \frac{\mu\left(\bar{K}\right)^{1-\mu}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\left(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k\right)^{2-2\mu}} \left[\begin{array}{c} -\frac{1}{\mu}(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}\phi^{\frac{1-\mu}{\mu}}\left(1+\gamma\beta\sigma_2\right) - \\ \left\{\gamma\left(1+\sigma_1\beta\right)\alpha - \phi^{\frac{1}{\mu}}\left(1+\gamma\beta\sigma_2\right)\right\} \\ \left\{(1-\mu)\left(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k\right)^{-\mu}\left(\frac{1}{\mu}\phi^{\frac{1-\mu}{\mu}}\bar{L}_t^k\right)\right\} \end{array}\right]$$

$$\frac{d\theta_t^{UD^*}}{d\phi} = -\frac{\mu\left(\bar{K}\right)^{1-\mu}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\left(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k\right)^{2-2\mu}} \left[\begin{array}{c} \left\{\frac{1}{\mu}(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{1-\mu}\phi^{\frac{1-\mu}{\mu}}\left(1+\gamma\beta\sigma_2\right)\right\} + \\ \left\{\gamma\left(1+\sigma_1\beta\right)\alpha - \phi^{\frac{1}{\mu}}\left(1+\gamma\beta\sigma_2\right)\right\} \\ \left\{(1-\mu)\left(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k\right)^{-\mu}\left(\frac{1}{\mu}\phi^{\frac{1-\mu}{\mu}}\bar{L}_t^k\right)\right\} \end{array}\right]$$

$$\frac{d\theta_t^{UD^*}}{d\phi} < 0$$

Following is the optimal proportion of total time of the unskilled adults to be spent in the source country:

$$\left(1-\delta^*\right) = \frac{\gamma\beta\sigma_2\left(\alpha+\phi^{\frac{1}{\mu}}\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\left(\alpha-1\right)}$$

Derivating $(1-\delta^*)$ with respect to α shows a negative influence of α on the optimal proportion of total time of the unskilled adults to be spent in the source country:

$$\frac{d\left(1-\delta^*\right)}{d\alpha} = \frac{\gamma\beta\sigma_2}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\left[\frac{\left(\alpha-1\right)\left(1\right)-\left(\alpha+\phi^{\frac{1}{\mu}}\right)\left(1\right)}{\left(\alpha-1\right)^2}\right]$$

$$\frac{d\left(1-\delta^*\right)}{d\alpha} = \frac{\gamma\beta\sigma_2}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\left[\frac{\alpha-1-\alpha-\phi^{\frac{1}{\mu}}}{\left(\alpha-1\right)^2}\right]$$

$$\frac{d\left(1-\delta^*\right)}{d\alpha} = \frac{\gamma\beta\sigma_2}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\left[\frac{-1-\phi^{\frac{1}{\mu}}}{\left(\alpha-1\right)^2}\right]$$

$$\frac{d\left(1-\delta^*\right)}{d\alpha} = -\frac{\gamma\beta\sigma_2\left(1+\phi^{\frac{1}{\mu}}\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\left(\alpha-1\right)^2}$$

$$\frac{d\left(1-\delta^*\right)}{d\alpha} < 0$$

Derivating $(1-\delta^*)$ with respect to ϕ shows a positive influence of ϕ on the optimal proportion of the unskilled worker's total time to be spent in the source country:

$$\frac{d\left(1-\delta^*\right)}{d\phi} = \frac{\gamma\beta\sigma_2}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\left(\alpha-1\right)}\left(\frac{1}{\mu}\phi^{\frac{1}{\mu}-1}\right)$$

$$\frac{d\left(1-\delta^{*}\right)}{d\phi}=\frac{\gamma\beta\sigma_{2}\phi^{\frac{1-\mu}{\mu}}}{\mu\left[1+\gamma+\gamma\beta\left(\sigma_{1}+\sigma_{2}\right)\right]\left(\alpha-1\right)}$$

$$\frac{d\left(1-\delta^{*}\right)}{d\phi}>0$$

13 Appendix G

In this appendix we will provide a proof determining the economy-wide child labor incidence and we also conduct a comparative static analysis on it. Using the time spent working by the child, $(1-e_t^*)$, and the total population of unskilled adults, L_t^U, the economy-wide child labor incidence, $\bar{L}_t^k$ is determined as follows:

$$\bar{L}_t^k=\left(1-e_t^*\right)L_t^U$$

$$e_t^*=\frac{\gamma\beta\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

Using the above value of e_t^*, we find $\bar{L}_t^k$ in the following way:

$$\left(1-e_t^*\right)=1-\frac{\gamma\beta\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

$$\left(1-e_t^*\right)=\frac{\left(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}}-\gamma\beta\sigma_1\alpha-\gamma\beta\sigma_1\phi^{\frac{1}{\mu}}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

$$\bar{L}_t^k=\left(1-e_t^*\right)L_t^U=\frac{\left[\left(1+\gamma+\gamma\beta\sigma_1+\gamma\beta\sigma_2-\gamma\beta\sigma_1\right)\phi^{\frac{1}{\mu}}-\gamma\beta\sigma_1\alpha\right]L_t^U}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

$$\bar{L}_t^k=\frac{\left[\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}}-\gamma\beta\sigma_1\alpha\right]L_t^U}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

where $\bar{L}_t^k$ represents the economy-wide child labor incidence.

Now we will be conducting a comparative static analysis of $\bar{L}_t^k$. Derivating $\bar{L}_t^k$ with respect to L_t^U reveals a positive impact of L_t^U on $\bar{L}_t^k$:

$$\frac{d\bar{L}_t^k}{dL_t^U} = \frac{\left[\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}} - \gamma\beta\sigma_1\alpha\right]}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

To check the sign of the numerator $\left[\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}} - \gamma\beta\sigma_1\alpha\right]$:

$$\left[\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}} - \gamma\beta\sigma_1\alpha\right] \gtreqless 0$$

$$\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}} \gtreqless \gamma\beta\sigma_1\alpha$$

$$\frac{\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}}}{\gamma\beta\sigma_1} \gtreqless \alpha$$

Since $\alpha = \frac{w_{UD}}{w_{US}}$ and $\phi^{\frac{1}{\mu}} = \frac{w_k}{w_{US}}$, the above inequality becomes as follows:

$$\frac{\left(1+\gamma+\gamma\beta\sigma_2\right)}{\gamma\beta\sigma_1}\frac{w_k}{w_{US}} \gtreqless \frac{w_{UD}}{w_{US}}$$

$$\frac{\left(1+\gamma+\gamma\beta\sigma_2\right)}{\gamma\beta\sigma_1}\frac{w_k}{w_{US}}w_{US} \gtreqless w_{UD}$$

$$\frac{\left(1+\gamma+\gamma\beta\sigma_2\right)}{\gamma\beta\sigma_1}w_k \gtreqless w_{UD}$$

According to the verification condition in appendix C:

$$w_{UD} < \frac{\left(1+\gamma+\gamma\beta\sigma_2\right)}{\gamma\beta\sigma_1}$$

Hence:

$$\left[\left(1+\gamma+\gamma\beta\sigma_2\right)\phi^{\frac{1}{\mu}} - \gamma\beta\sigma_1\alpha\right] > 0$$

$$\frac{d\bar{L}_t^k}{dL_t^U} > 0$$

The fall in the total population of unskilled adults reduces $\bar{L}_t^k$ because the increase in unskilled migrants increases the optimal amount of remittances, thereby reducing child's time spent on working. Derivating $\theta_t^{UD^*}$ with respect to M, reveals a positive impact of M on $\theta_t^{UD^*}$:

$$\frac{d\theta_t^{UD^*}}{dM} = \frac{-(1-\mu)(L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{-(1-\mu)-1}(-1)\mu\left(\bar{K}\right)^{1-\mu}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}\left[\gamma(1+\sigma_1\beta)\alpha - \phi^{\frac{1}{\mu}}\left(1+\gamma\beta\sigma_2\right)\right]$$

$$\frac{d\theta_t^{UD^*}}{dM} = \frac{(1-\mu)\mu\left(\bar{K}\right)^{1-\mu}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right](L_t^U - M + \phi^{\frac{1}{\mu}}\bar{L}_t^k)^{2-\mu}}\left[\gamma(1+\sigma_1\beta)\alpha - \phi^{\frac{1}{\mu}}\left(1+\gamma\beta\sigma_2\right)\right]$$

$$\frac{d\theta_t^{UD^*}}{dM} > 0$$

Derivating $\bar{L}_t^k$ with respect to α reveals a negative impact of α on $\bar{L}_t^k$:

$$\frac{d\bar{L}_t^k}{d\alpha} = -\frac{\gamma\beta\sigma_1 L_t^U}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}}$$

$$\frac{d\bar{L}_t^k}{d\alpha} < 0$$

$$\bar{L}_t^k = \frac{\left(1+\gamma+\gamma\beta\sigma_2\right)L_t^U}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]} - \frac{-\gamma\beta\sigma_1\alpha L_t^U\phi^{-\frac{1}{\mu}}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]}$$

Derivating $\bar{L}_t^k$ with respect to ϕ reveals a positive impact of ϕ on $\bar{L}_t^k$:

$$\frac{d\bar{L}_t^k}{d\phi} = -\frac{\left(-\frac{1}{\mu}\right)\gamma\beta\sigma_1\alpha L_t^U}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1+\mu}{\mu}}}$$

$$\frac{d\bar{L}_t^k}{d\phi} = \frac{\gamma\beta\sigma_1\alpha L_t^U}{\mu\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1+\mu}{\mu}}}$$

$$\frac{d\bar{L}_t^k}{d\phi} > 0$$

14 Appendix H

In this appendix, we derive the level of human capital of the unskilled workers' child in time period $t+1$ and then show a detailed comparative static analysis of H_{t+1}. To find H_{t+1},plug in e_t^* and $(1-\delta^*)$ in the human capital function:

$$H_{t+1} = \lambda H_t\left(e_t^*\right)^{\sigma_1}(1-\delta^*)^{\sigma_2}$$

$$H_{t+1} = \lambda H_t \left[\frac{\gamma\beta\sigma_1 \left(\alpha + \phi^{\frac{1}{\mu}}\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]\phi^{\frac{1}{\mu}}} \right]^{\sigma_1} \left[\frac{\gamma\beta\sigma_2 \left(\alpha + \phi^{\frac{1}{\mu}}\right)}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right](\alpha-1)} \right]^{\sigma_2}$$

$$H_{t+1} = \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}}$$

Thus, following is the level of human capital of the unskilled workers' child in time period $t+1$:

$$H_{t+1} = \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} (\alpha-1)^{\sigma_2}}$$

Now we will conduct a comparative static analysis of H_{t+1}. Derivating H_{t+1} with respect to α_t shows a non-linear impact of α on H_{t+1}:

$$\frac{dH_{t+1}}{d\alpha} = \frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}} \left[\frac{\begin{array}{c} (\alpha-1)^{\sigma_2} \left(\sigma_1+\sigma_2\right) \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2-1} (1) \\ - \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \sigma_2 (\alpha-1)^{\sigma_2-1} \end{array}}{(\alpha-1)^{2\sigma_2}} \right]$$

$$\frac{dH_{t+1}}{d\alpha} = \frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} (\alpha-1)^{2\sigma_2}} \left(\left[(\alpha-1)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \right] \left[\frac{\left(\sigma_1+\sigma_2\right)}{\left(\alpha + \phi^{\frac{1}{\mu}}\right)} - \frac{\sigma_2}{(\alpha-1)} \right] \right)$$

$$\frac{dH_{t+1}}{d\alpha} = \frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} (\alpha-1)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} (\alpha-1)^{2\sigma_2}} \left[\frac{\left(\sigma_1+\sigma_2\right)}{\left(\alpha + \phi^{\frac{1}{\mu}}\right)} - \frac{\sigma_2}{(\alpha-1)} \right]$$

$$\frac{dH_{t+1}}{d\alpha} = \frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} (\alpha-1)^{\sigma_2}} \left[\frac{\left(\sigma_1+\sigma_2\right)}{\left(\alpha + \phi^{\frac{1}{\mu}}\right)} - \frac{\sigma_2}{(\alpha-1)} \right]$$

The first term $\frac{\lambda(\gamma\beta\sigma_1)^{\sigma_1}(\gamma\beta\sigma_2)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{\sigma_2}}$ is positive. Therefore, the sign of the derivative depends upon the sign of the second term $\left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)} - \frac{\sigma_2}{(\alpha-1)}\right]$. Checking the sign of the second term:

$$\left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\frac{\sigma_2}{(\alpha-1)}\right]\gtreqless 0$$

$$\frac{(\sigma_1+\sigma_2)(\alpha-1)-\sigma_2\left(\alpha+\phi^{\frac{1}{\mu}}\right)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)(\alpha-1)}\gtreqless 0$$

$$\frac{\alpha\sigma_2+\alpha\sigma_1-\sigma_2-\sigma_1-\alpha\sigma_2-\sigma_2\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)(\alpha-1)}\gtreqless 0$$

$$\frac{\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)(\alpha-1)}\gtreqless 0$$

$$\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}}\gtreqless 0$$

$$\frac{dH_{t+1}}{d\alpha}=0 \text{ if } (\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}})=0$$

$$\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}}=0$$

$$\alpha\sigma_1=\sigma_2+\sigma_1+\sigma_2\phi^{\frac{1}{\mu}}$$

$$\alpha=\frac{\sigma_2+\sigma_1+\sigma_2\phi^{\frac{1}{\mu}}}{\sigma_1}$$

$$\alpha=\frac{\sigma_2+\sigma_2\phi^{\frac{1}{\mu}}}{\sigma_1}+\frac{\sigma_1}{\sigma_1}$$

$$\alpha=\frac{\sigma_2\left(1+\phi^{\frac{1}{\mu}}\right)}{\sigma_1}+1$$

$$\alpha=1+\frac{\sigma_2}{\sigma_1}\left(1+\phi^{\frac{1}{\mu}}\right)=\alpha_A$$

$$\frac{dH_{t+1}}{d\alpha}>0 \text{ if } (\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}})>0$$

$$\frac{dH_{t+1}}{d\alpha}>0 \text{ if } \alpha>\left[1+\frac{\sigma_2}{\sigma_1}\left(1+\phi^{\frac{1}{\mu}}\right)\right]$$

$$\frac{dH_{t+1}}{d\alpha}>0 \text{ if } \alpha>\alpha_A$$

$$\frac{dH_{t+1}}{d\alpha}<0 \text{ if } (\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}})<0$$

$$\frac{dH_{t+1}}{d\alpha} < 0 \text{ if } \alpha < \left[1 + \frac{\sigma_2}{\sigma_1}\left(1 + \phi^{\frac{1}{\mu}}\right)\right]$$

$$\frac{dH_{t+1}}{d\alpha} < 0 \text{ if } \alpha < \alpha_A$$

Threshold value:

$$\alpha = 1 + \frac{\sigma_2}{\sigma_1}\left(1 + \phi^{\frac{1}{\mu}}\right) = \alpha_A$$

Special case if $\sigma_2 = \sigma_1$:

$$\alpha = 1 + \frac{\sigma_2}{\sigma_2}\left(1 + \phi^{\frac{1}{\mu}}\right)$$

$$\alpha = 1 + 1\left(1 + \phi^{\frac{1}{\mu}}\right)$$

$$\alpha = 1 + 1 + \phi^{\frac{1}{\mu}}$$

$$\alpha = 2 + \phi^{\frac{1}{\mu}} = \alpha_B$$

$$\frac{dH_{t+1}}{d\alpha} > 0 \text{ if } \alpha > \left(2 + \phi^{\frac{1}{\mu}}\right)$$

$$\frac{dH_{t+1}}{d\alpha} > 0 \text{ if } \alpha > \alpha_B$$

$$\frac{dH_{t+1}}{d\alpha} < 0 \text{ if } \alpha < \left(2 + \phi^{\frac{1}{\mu}}\right)$$

$$\frac{dH_{t+1}}{d\alpha} < 0 \text{ if } \alpha < \alpha_B$$

$$\frac{dH_{t+1}}{d\alpha} = 0 \text{ if } \alpha = \left(2 + \phi^{\frac{1}{\mu}}\right)$$

$$\frac{dH_{t+1}}{d\alpha} = 0 \text{ if } \alpha = \alpha_B$$

Derivating H_{t+1} with respect to ϕ shows a negative impact of ϕ on H_{t+1}:

$$\frac{dH_{t+1}}{d\phi} = \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1 + \gamma + \gamma\beta\left(\sigma_1 + \sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\alpha - 1\right)^{\sigma_2}} \left[\frac{\begin{array}{c} \phi^{\frac{\sigma_1}{\mu}} \left(\sigma_1 + \sigma_2\right) \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2-1} \left(\frac{1}{\mu}\phi^{\frac{1}{\mu}-1}\right) \\ - \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \left(\frac{\sigma_1}{\mu}\phi^{\frac{\sigma_1}{\mu}-1}\right) \end{array}}{\left(\phi^{\frac{\sigma_1}{\mu}}\right)^2}\right]$$

$$\frac{dH_{t+1}}{d\phi}=\frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\alpha-1\right)^{\sigma_2}\left(\phi^{\frac{\sigma_1}{\mu}}\right)^2}\left[\begin{array}{c}\left(\sigma_1+\sigma_2\right)\left(\frac{1}{\mu}\phi^{\frac{\sigma_1-\mu+1}{\mu}}\right)\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2-1}\\-\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}\left(\frac{\sigma_1}{\mu}\phi^{\frac{\sigma_1-\mu}{\mu}}\right)\end{array}\right]$$

$$\frac{dH_{t+1}}{d\phi}=\frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\alpha-1\right)^{\sigma_2}\left(\phi^{\frac{\sigma_1}{\mu}}\right)^2}\left[\left(\frac{1}{\mu}\phi^{\frac{\sigma_1-\mu}{\mu}}\right)\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}\left(\frac{(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\sigma_1\right)\right]$$

$$\frac{dH_{t+1}}{d\phi}=\frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\mu\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\alpha-1\right)^{\sigma_2}\left(\phi\right)^{\frac{2\sigma_1-(\sigma_1-\mu)}{\mu}}}\left[\left(\frac{(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\sigma_1\right)\right]$$

$$\frac{dH_{t+1}}{d\phi}=\frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\mu\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\alpha-1\right)^{\sigma_2}\left(\phi\right)^{\frac{\sigma_1+\mu}{\mu}}\left(\alpha+\phi^{\frac{1}{\mu}}\right)}\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}-\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right]$$

$$\frac{dH_{t+1}}{d\phi}=\frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\mu\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\alpha-1\right)^{\sigma_2}\left(\phi\right)^{\frac{\sigma_1+\mu}{\mu}}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{1-\sigma_1-\sigma_2}}\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}-\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right]$$

The first term is positive. Hence, the sign of the derivative depends on the sign of the term $\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}-\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right]$. Checking the sign of the second term:

$$\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}-\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right]\gtreqless 0$$

$$[\sigma_2\phi^{\frac{1}{\mu}}+\sigma_1\phi^{\frac{1}{\mu}}-\sigma_1\alpha-\sigma_1\phi^{\frac{1}{\mu}}]\gtreqless 0$$

$$\left(\sigma_2\phi^{\frac{1}{\mu}}-\sigma_1\alpha\right)\gtreqless 0$$

Since $\alpha=\frac{w_{UD}}{w_{US}}$ and $\phi^{\frac{1}{\mu}}=\frac{w_k}{w_{US}}$, the above inequality becomes as follows:

$$\left(\sigma_2\frac{w_k}{w_{US}}-\sigma_1\frac{w_{UD}}{w_{US}}\right)\gtreqless 0$$

$$\left(\frac{\sigma_2 w_k-\sigma_1 w_{UD}}{w_{US}}\right)\gtreqless 0$$

$$(\sigma_2 w_k-\sigma_1 w_{UD})\gtreqless 0$$

Since, $\sigma_1<1$ and $\sigma_2<1$ so:

$$(\sigma_2 w_k-\sigma_1 w_{UD})<0$$

Hence,

$$\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}} - \sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right] < 0 \text{ so } \frac{dH_{t+1}}{d\phi} < 0$$

15 Appendix I

In this appendix, we provide a detailed proof of how the level of human capital of all the children in the economy in time period $t+1$ is calculated and performing a comparative static analysis on it. Then we will show how this helps us to find the economic growth in the source country. The level of human capital of all the children in the economy in time period $t+1$ (i.e., H_{t+1}^E) is a weighted average of H_{t+1}^S and H_{t+1}^U as shown below:

$$H_{t+1}^E = H_{t+1}^S\left(\frac{L_t^S}{N_t}\right) + H_{t+1}^U\left(\frac{L_t^U}{N_t}\right) \tag{1}$$

The level of human capital of the unskilled workers' child in time period $t+1$ is as shown:

$$H_{t+1}^U = \frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{\sigma_2}} \tag{2}$$

The level of human capital of the skilled workers' child in time period $t+1$ is as shown:

$$H_{t+1}^S = \lambda H_t \tag{3}$$

The total population of the source country is N_t which is split up into the skilled adults L_t^S and the unskilled adults L_t^U as shown:

$$N_t = L_t^S + L_t^U \tag{4}$$

Plugging (2), (3) and (4) into (1), we arrive at H_{t+1}^E as follows:

$$H_{t+1}^E = \lambda H_t\left(\frac{L_t^S}{L_t^S+L_t^U}\right) + \left(\frac{\lambda H_t\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{\sigma_2}}\right)\left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

Now we will be conducting a comparative static analysis of H_{t+1}^E with respect to ϕ, α and L_t^U. Derivating H_{t+1}^E with respect to ϕ shows a negative impact of ϕ on H_{t+1}^E as follows:

$$\frac{dH_{t+1}^E}{d\phi} = \frac{\lambda H_t (\gamma\beta\sigma_1)^{\sigma_1} (\gamma\beta\sigma_2)^{\sigma_2}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2}} \left[\frac{\begin{array}{c} \phi^{\frac{\sigma_1}{\mu}} (\sigma_1+\sigma_2) \left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2-1} \left(\frac{1}{\mu}\phi^{\frac{1}{\mu}-1}\right) \\ -\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \left(\frac{\sigma_1}{\mu}\phi^{\frac{\sigma_1}{\mu}-1}\right) \end{array}}{\left(\phi^{\frac{\sigma_1}{\mu}}\right)^2} \right] \left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

$$\frac{dH_{t+1}^E}{d\phi} = \frac{\lambda H_t (\gamma\beta\sigma_1)^{\sigma_1} (\gamma\beta\sigma_2)^{\sigma_2} L_t^U}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2} \left(\phi^{\frac{\sigma_1}{\mu}}\right)^2 \left(L_t^S+L_t^U\right)} \left[\begin{array}{c} (\sigma_1+\sigma_2) \left(\frac{1}{\mu}\phi^{\frac{\sigma_1-\mu+1}{\mu}}\right) \left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2-1} \\ -\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \left(\frac{\sigma_1}{\mu}\phi^{\frac{\sigma_1-\mu}{\mu}}\right) \end{array} \right]$$

$$\frac{dH_{t+1}^E}{d\phi} = \frac{\lambda H_t (\gamma\beta\sigma_1)^{\sigma_1} (\gamma\beta\sigma_2)^{\sigma_2} L_t^U}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2} \left(\phi^{\frac{\sigma_1}{\mu}}\right)^2 \left(L_t^S+L_t^U\right)} \left[\begin{array}{c} \left(\frac{1}{\mu}\phi^{\frac{\sigma_1-\mu}{\mu}}\right) \left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} \\ \left(\frac{(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)} - \sigma_1\right) \end{array} \right]$$

$$\frac{dH_{t+1}^E}{d\phi} = \frac{\lambda H_t (\gamma\beta\sigma_1)^{\sigma_1} (\gamma\beta\sigma_2)^{\sigma_2} \left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\mu[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2} (\phi)^{\frac{2\sigma_1-(\sigma_1-\mu)}{\mu}}} \left[\left(\frac{(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)} - \sigma_1\right)\right] \left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

$$\frac{dH_{t+1}^E}{d\phi} = \frac{\lambda H_t (\gamma\beta\sigma_1)^{\sigma_1} (\gamma\beta\sigma_2)^{\sigma_2} \left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\mu[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2} (\phi)^{\frac{\sigma_1+\mu}{\mu}} \left(\alpha+\phi^{\frac{1}{\mu}}\right)} \left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}} - \sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right] \left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

$$\frac{dH_{t+1}^E}{d\phi} = \frac{\lambda H_t (\gamma\beta\sigma_1)^{\sigma_1} (\gamma\beta\sigma_2)^{\sigma_2}}{\mu[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2} (\alpha-1)^{\sigma_2} (\phi)^{\frac{\sigma_1+\mu}{\mu}} \left(\alpha+\phi^{\frac{1}{\mu}}\right)^{1-\sigma_1-\sigma_2}} \left[\begin{array}{c} (\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}} \\ -\sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right) \end{array} \right] \left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

The sign of the derivative depends on the sign of the term $\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}} - \sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right]$. Checking the sign of this term:

$$\left[(\sigma_1+\sigma_2)\phi^{\frac{1}{\mu}} - \sigma_1\left(\alpha+\phi^{\frac{1}{\mu}}\right)\right] \gtreqless 0$$

$$\left[\sigma_2\phi^{\frac{1}{\mu}} + \sigma_1\phi^{\frac{1}{\mu}} - \sigma_1\alpha - \sigma_1\phi^{\frac{1}{\mu}}\right] \gtreqless 0$$

$$\left(\sigma_2\phi^{\frac{1}{\mu}} - \sigma_1\alpha\right) \gtreqless 0$$

Since $\alpha = \frac{w_{UD}}{w_{US}}$ and $\phi^{\frac{1}{\mu}} = \frac{w_k}{w_{US}}$, the above inequality becomes as follows:

$$\left(\sigma_2 \frac{w_k}{w_{US}} - \sigma_1 \frac{w_{UD}}{w_{US}}\right) \gtreqless 0$$

$$\left(\frac{\sigma_2 w_k - \sigma_1 w_{UD}}{w_{US}}\right) \gtreqless 0$$

$$(\sigma_2 w_k - \sigma_1 w_{UD}) \gtreqless 0$$

Since, $\sigma_1 < 1$ and $\sigma_2 < 1$ so:

$$(\sigma_2 w_k - \sigma_1 w_{UD}) < 0$$

Hence,

$$\left[(\sigma_1 + \sigma_2)\phi^{\frac{1}{\mu}} - \sigma_1\left(\alpha + \phi^{\frac{1}{\mu}}\right)\right] < 0 \text{ so } \frac{dH_{t+1}^E}{d\phi} < 0$$

Derivating H_{t+1}^E with respect to α shows a non-linear impact of α on H_{t+1}^E as follows:

$$\frac{dH_{t+1}^E}{d\alpha} = \frac{\lambda\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}} \left[\frac{\begin{array}{c}(\alpha-1)^{\sigma_2}\left(\sigma_1+\sigma_2\right)\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2-1}(1)\\ -\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}\sigma_2(\alpha-1)^{\sigma_2-1}\end{array}}{(\alpha-1)^{2\sigma_2}}\right]\left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

$$\frac{dH_{t+1}^E}{d\alpha} = \frac{\lambda\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{2\sigma_2}}\left(\begin{array}{c}\left[(\alpha-1)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}\right]\\ \left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\frac{\sigma_2}{(\alpha-1)}\right]\end{array}\right)\left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

$$\frac{dH_{t+1}^E}{d\alpha} = \frac{\lambda\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}(\alpha-1)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{2\sigma_2}}\left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\frac{\sigma_2}{(\alpha-1)}\right]\left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

$$\frac{dH_{t+1}^E}{d\alpha} = \frac{\lambda\left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{\sigma_2}}\left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\frac{\sigma_2}{(\alpha-1)}\right]\left(\frac{L_t^U}{L_t^S+L_t^U}\right)$$

The term $\left(\frac{\lambda(\gamma\beta\sigma_1)^{\sigma_1}(\gamma\beta\sigma_2)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{\sigma_2}}\right)\left(\frac{L_t^U}{L_t^S+L_t^U}\right)$is positive. Therefore, the sign of the derivative depends upon the sign of the term $\left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\frac{\sigma_2}{(\alpha-1)}\right]$. Checking the sign of the second term:

$$\left[\frac{(\sigma_1+\sigma_2)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)}-\frac{\sigma_2}{(\alpha-1)}\right] \gtreqless 0$$

$$\frac{(\sigma_1+\sigma_2)(\alpha-1)-\sigma_2\left(\alpha+\phi^{\frac{1}{\mu}}\right)}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)(\alpha-1)} \gtreqless 0$$

$$\frac{\alpha\sigma_2+\alpha\sigma_1-\sigma_2-\sigma_1-\alpha\sigma_2-\sigma_2\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)(\alpha-1)} \gtreqless 0$$

$$\frac{\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}}}{\left(\alpha+\phi^{\frac{1}{\mu}}\right)(\alpha-1)} \gtreqless 0$$

$$\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}} \gtreqless 0$$

$$\frac{dH_{t+1}^E}{d\alpha}=0 \text{ if } (\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}})=0$$

$$\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}}=0$$

$$\alpha\sigma_1=\sigma_2+\sigma_1+\sigma_2\phi^{\frac{1}{\mu}}$$

$$\alpha=\frac{\sigma_2+\sigma_1+\sigma_2\phi^{\frac{1}{\mu}}}{\sigma_1}$$

$$\alpha=\frac{\sigma_2+\sigma_2\phi^{\frac{1}{\mu}}}{\sigma_1}+\frac{\sigma_1}{\sigma_1}$$

$$\alpha=\frac{\sigma_2\left(1+\phi^{\frac{1}{\mu}}\right)}{\sigma_1}+1$$

$$\alpha=1+\frac{\sigma_2}{\sigma_1}\left(1+\phi^{\frac{1}{\mu}}\right)=\alpha_A$$

$$\frac{dH_{t+1}^E}{d\alpha}>0 \text{ if } (\alpha\sigma_1-\sigma_2-\sigma_1-\sigma_2\phi^{\frac{1}{\mu}})>0$$

$$\frac{dH_{t+1}^{E}}{d\alpha} > 0 \text{ if } \alpha > \left[1 + \frac{\sigma_2}{\sigma_1}\left(1 + \phi^{\frac{1}{\mu}}\right)\right]$$

$$\frac{dH_{t+1}^{E}}{d\alpha} > 0 \text{ if } \alpha > \alpha_A$$

$$\frac{dH_{t+1}^{E}}{d\alpha} < 0 \text{ if } (\alpha\sigma_1 - \sigma_2 - \sigma_1 - \sigma_2\phi^{\frac{1}{\mu}}) < 0$$

$$\frac{dH_{t+1}^{E}}{d\alpha} < 0 \text{ if } \alpha < \left[1 + \frac{\sigma_2}{\sigma_1}\left(1 + \phi^{\frac{1}{\mu}}\right)\right]$$

$$\frac{dH_{t+1}^{E}}{d\alpha} < 0 \text{ if } \alpha < \alpha_A$$

Derivating H_{t+1}^{E} with respect to L_t^U shows a negative impact of L_t^U on H_{t+1}^{E} as follows:

$$\frac{dH_{t+1}^{E}}{dL_t^U} = \frac{-\lambda H_t L_t^S}{(L_t^S + L_t^U)^2} + \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1 + \gamma + \gamma\beta\left(\sigma_1 + \sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha - 1)^{\sigma_2}}\left[\frac{(L_t^S + L_t^U)(1) - \left(L_t^U\right)(1)}{(L_t^S + L_t^U)^2}\right]$$

$$\frac{dH_{t+1}^{E}}{dL_t^U} = \frac{-\lambda H_t L_t^S}{(L_t^S + L_t^U)^2} + \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2} L_t^S}{\left[1 + \gamma + \gamma\beta\left(\sigma_1 + \sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha - 1)^{\sigma_2}(L_t^S + L_t^U)^2}$$

$$\frac{dH_{t+1}^{E}}{dL_t^U} = \frac{L_t^S}{(L_t^S + L_t^U)^2}\left[-\lambda H_t + \frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1}\left(\gamma\beta\sigma_2\right)^{\sigma_2}\left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1 + \gamma + \gamma\beta\left(\sigma_1 + \sigma_2\right)\right]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha - 1)^{\sigma_2}}\right]$$

In the above expression, $\lambda H_t = H_{t+1}^S$ while $\frac{\lambda H_t(\gamma\beta\sigma_1)^{\sigma_1}(\gamma\beta\sigma_2)^{\sigma_2}\left(\alpha+\phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{[1+\gamma+\gamma\beta(\sigma_1+\sigma_2)]^{\sigma_1+\sigma_2}\left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1}(\alpha-1)^{\sigma_2}} = H_{t+1}^U$. $H_{t+1}^S > H_{t+1}^U$ because in the skilled sector children devote their entire time to education. Moreover, the skilled parents spend all their time in the source country. Henceforth, the children of the skilled adults end up accumulating greater levels of human capital. Thus, when a larger term is deducted from a smaller term, the overall impact is negative:

$$\frac{dH_{t+1}^{E}}{dL_t^U} < 0$$

Now we will find the growth rate of human capital. The growth of human capital can be calculated as follows:

$$g_h = \frac{H_{t+1}^E}{H_t} - 1$$

Since H_{t+1}^E is as shown:

$$H_{t+1}^E = \lambda H_t \left(\frac{L_t^S}{L_t^S + L_t^U} \right) + \left(\frac{\lambda H_t \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \right) \left(\frac{L_t^U}{L_t^S + L_t^U} \right)$$

Plugging H_{t+1}^E into the expression of g_h we arrive at the following:

$$g_h = \left[\lambda \left(\frac{L_t^S}{L_t^S + L_t^U} \right) + \left(\frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \right) \left(\frac{L_t^U}{L_t^S + L_t^U} \right) \right] - 1$$

$$(1+g_h) = \lambda \left(\frac{L_t^S}{L_t^S + L_t^U} \right) + \left(\frac{\lambda \left(\gamma\beta\sigma_1\right)^{\sigma_1} \left(\gamma\beta\sigma_2\right)^{\sigma_2} \left(\alpha + \phi^{\frac{1}{\mu}}\right)^{\sigma_1+\sigma_2}}{\left[1+\gamma+\gamma\beta\left(\sigma_1+\sigma_2\right)\right]^{\sigma_1+\sigma_2} \left(\phi^{\frac{1}{\mu}}\right)^{\sigma_1} \left(\alpha-1\right)^{\sigma_2}} \right) \left(\frac{L_t^U}{L_t^S + L_t^U} \right)$$

The growth of human capital is also showing the growth of output. To prove this, let us have a look at the total output in the source country, Y_t, which is split up into the output of the skilled sector, $Y_t^{Skilled}$, and output of the unskilled sector, $Y_t^{Unskilled}$:

$$Y_t = Y_t^{Skilled} + \varphi Y_t^{Unskilled}$$

where $\varphi \in (0,1)$. The output of the skilled sector, $Y_t^{Skilled}$, is determined by human capital of the skilled workers, H_t^S, as follows:

$$Y_t^S = H_t^S$$

The output of the unskilled sector, $Y_t^{Unskilled}$, comprises of the output produced by two further sectors; the adult sector and child sector as follows:

$$Y_t^{Unskilled} = \rho Y_t^k + Y_t^U$$

where $\rho \in (0, 1)$, Y_t^k is the output produced by chilldren and Y_t^U is the output produced by the unskilled adults. Therefore, total output in the source country is as follows:

$$Y_t = H_t^S + \varphi \left(\rho Y_t^k + Y_t^U \right)$$

As is apparent from the above equation, output will grow at the rate of growth of human capital of the skilled workers.

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