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The Theory of the Natural Resource Curse: A Political Economy View

Sadia Hussain
Waqar Wadho

Centre for Research in Economics and Business
Lahore School of Economics

Centre for Research in Economics and Business (CREB)

Lahore School of Economics

Intersection Main Boulevard Phase VI, DHA and Burki Road
Lahore 53200, Pakistan
Tel: 042-36561230; 042-36560936
Email: creb@lahoreschool.edu.pk

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Sadia Hussain
Teaching and Research Fellow, Lahore School of Economics

Waqar Wadho
Assistant Professor, Lahore School of Economics

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Lahore School of Economics
Intersection of Main Boulevard, Phase VI, DHA, and Burki Road
Lahore 53200, Pakistan
Tel.: +92 42 3656 1230
www.creb.org.pk
creb@lahoreschool.edu.pk

Price: PRs100

Preface

The Centre for Research in Economics and Business (CREB) was established in 2007 to conduct policy-oriented research with a rigorous academic perspective on key development issues facing Pakistan. In addition, CREB (i) facilitates and coordinates research by faculty at the Lahore School of Economics, (ii) hosts visiting international scholars undertaking research on Pakistan, and (iii) administers the Lahore School's postgraduate program leading to the MPhil and PhD degrees.

An important goal of CREB is to promote public debate on policy issues through conferences, seminars, and publications. In this connection, CREB organizes the Lahore School's Annual Conference on the Management of the Pakistan Economy, the proceedings of which are published in a special issue of the Lahore Journal of Economics.

The CREB Working Paper Series was initiated in 2008 to bring to a wider audience the research being carried out at the Centre. It is hoped that these papers will promote discussion on the subject and contribute to a better understanding of economic and business processes and development issues in Pakistan. Comments and feedback on these papers are welcome.

Abstract

This study revisits the conditionality of the natural resource curse using ethnic polarization as the center of analysis. It argues that the resource curse is triggered in ethnically polarized societies when productive resources are reallocated toward rent-seeking activities. The study builds a theoretical model in a general equilibrium setting to explain how the elite expropriate natural resource rents in the economy. The elite decision problem comprises (i) the decision to become an elite actor through a marginal benefit–cost analysis, followed by (ii) a decision concerning the optimal level of rent. The results show how ethnically polarized societies become trapped in a high-corruption equilibrium when the returns from rent seeking are raised. Conversely, in ethnically homogenous societies, a low equilibrium acts as a conduit for a lower concentration of political power.

The Theory of the Natural Resource Curse: A Political Economy View

1. Introduction

While many economists have questioned the role of natural resources in economic growth, following the departure from trade protectionist policies, there is no unifying framework that explains the extent to which natural resources affect growth. To address this, one must look at empirical regularities. While economic intuition points to the additive role of natural resources as an input to production, the empirical evidence suggests otherwise.[1]

We have seen that countries abundant in natural resources – namely, mineral oil and natural gas – tend to grow more slowly than resource-scarce economies. In this context, Nigeria is often compared with Norway to illustrate the 'natural resource curse'. Despite earning US$200 million in foreign exchange from oil exports between 1970 and 1990, Nigeria was unable to channel these gains into higher growth and development indicators. Over the last ten years, the country has experienced a modest increase in real GDP growth rates and lags well behind the world average.[2] Like Nigeria, Norway is well endowed with mineral oil and natural gas, but it has not fallen prey to the natural resource curse. The evidence suggests that attributing lower growth rates to natural resources can yield misleading causal correlation. This study addresses the missing piece of the puzzle by introducing corruption as a variable through ethnic polarization.

Economic theory on the natural resource curse has transitioned from a market-based view to an institutional one. Sachs and Warner (1995) were among the first to identify the growth-retarding aspect of natural resources. They link natural resource flows to exchange rate appreciation, which leads to the reallocation of productive resources from manufacturing to the natural resources sector. Committing funds to the latter leads to allocative inefficiencies by drawing resources away from

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[1] See, for example, Sachs and Warner (1995) and Sala-i-Martin and Subramanian (2003).

[2] Based on data available from the World Bank at http://data.worldbank.org/

the economy's more productive sectors – the phenomenon termed 'Dutch disease'. However, Bulte, Damania and Deacon (2005) argue that this hypothesis lacks empirical support.

Other market-based theories also rely on the 'crowding-out' effect to explain the resource curse effect. Resource booms divert resources away from productive activities – such as learning by doing, the accumulation of human capital and entrepreneurial talent – to rent-seeking activities (Gylfason & Zoega, 2006; Tornell & Lane, 1999; Torvik, 2002). The political economy literature tends to debunk market theories, although political economists, too, are divided on this. Nonetheless, the consensus centers on the role of institutional forces, where natural resources are accompanied by poor governance, corrupt bureaucracies and the expropriation of resource rents.

Proponents of the political view argue that inclusive institutions allow resource abundance to serve as a positive factor, whereas extractive institutions tend to divert productive resources toward rent-seeking activities, thereby triggering the resource curse (Leite & Weidmann, 1999; Ross, 1999; Mehlum, Moene & Torvik, 2006). Karl (1997) shows that natural resources can change the nexus of political power between the public and the elite, thus perpetuating a vicious cycle between resource rents and extractive institutions.

Most theoretical frameworks in this field model institutions in two ways: (i) by explicitly bringing the government into the analysis or (ii) using government policy as the equilibrium outcome of rent-seeking activities (Deacon, 2011). This study employs a similar approach by casting policy as an outcome of corruption from rent seeking. It is important to identity the causal links between elite power plays in resource-endowed societies. We introduce ethnic polarization as the lynchpin of our analysis in explaining the natural resource curse.

Contributions in this field center on either civil conflict or corruption. The first thesis, put forward by studies such as Reynal-Querol (2002), contends that ethnic polarization triggers civil conflict and political instability through power struggles played out among different factions of society. Caselli (2006) examines the conditionality of the resource curse, showing that resource rents raise the discount rate of the governing elite by diverting resources toward rent seeking vis-à-vis lowering investment in public goods. Were the incumbent actors to invest enough in productive capital, thereby preventing power struggles, the fate of the economy could be reversed.

On the empirical side, Collier, Hoeffler and Rohner (2009) explore the possible causes of civil conflict by testing the feasibility hypothesis: that is, conflict will occur whenever there are financial gains associated with political instability. In the presence of ethnic factions, civil conflict is associated with profits, although this relationship is nonmonotonic. Conflict-based theories are often downplayed for lack of empirical support and flawed econometric methodologies. Indeed, not all ethnically polarized societies are necessarily subject to civil conflict.

The second argument uses the corruption factor to connect natural resources and low growth rates. Corruption imposes certain costs on society, including lower investment, the diversion of productive resources away from growth-inducing sectors to rent-seeking activities and the uncertainty associated with corrupt contracts. Mauro (1998), for instance, tests this argument empirically and finds that education spending bears the brunt of increased corruption in an economy.

The balanced-growth models show that the elite must decide whether to invest in political or human capital. Corrupt government intervention creates deviations between market and shadow prices – these are termed 'rents'. The incentive to invest in political capital deters investment in human capital, which is seen as the engine of growth in modern economics. Thus, an economy can experience multiple equilibria depending on the level of investment in political capital (Ehrlich & Lui, 1999; Wadho, 2014).

When political power is concentrated among the elite and their share of resource rents is large enough to secure a coalition of elite subgroups, it becomes possible to explain the empirical irregularities observed in resource-rich countries. An ethnically polarized economy is an ideal breeding ground for corruption in the presence of natural resource rents. In this case, we define 'ethnic polarization' as being a substantial difference in the preferences of a large ethnic majority and those of a large ethnic minority.

We also borrow the concept of 'social distance' from sociology to understand the political dynamics of rent seeking in an ethnically polarized society. Karakayali (2009) defines four categories of social distance: (i) affective, (ii) normative, (iii) interactive and (iv) cultural and habitual. Ethnic polarization drives a wedge between the normative and cultural values of different groups. Greater cultural and social heterogeneity leads to lower levels of trust among individuals and to higher transaction costs among groups.

Where studies such as Landa (1981) and Boettke, Coyne and Leeson (2008) use social distance to understand firm behavior and trade, we transplant the concept of social distance and transaction costs in a political framework. Agents closer to the center have similar values and cultures and can therefore build an elite network. Given the lower transaction costs in their circle, the elite have an insider advantage. This creates natural barriers to entry in the bureaucratic process, enabling a 'chosen few' to penetrate the elite circle.

The paper proceeds as follows. Section 2 provides a conceptual framework defining each component of the economy, including the size of the elite. Section 3 presents the results of our model in a steady-state equilibrium, based on which we determine the impact of rent seeking on growth. Section 4 concludes the study.

2. Conceptual Framework

This study relies on a neoclassical general equilibrium framework to link resource rents with ethnic polarization. We assume a closed economy with an infinite time horizon, a total population of N and a given stock of natural reserves denoted by S_0. The total population is divided into two distinct ethnic groups: an ethnic majority and a collective ethnic minority. There is no population growth, which means that the ratio of the ethnic majority to ethnic minorities remains constant over time.

θ is a fixed parameter that represents the degree of ethnic polarization: the higher the value of θ, the greater the divide between the cultural and normative values of both ethnic groups. The size of the elite and nonelite in this economy is denoted by sN and $(1 - s)N$, respectively, where s lies between 0 and 1. Agents in either group have differentiated access to political capital. This is defined as access to political power depending on the agent's social distance from the center, denoted by z_i. Only those agents with a threshold level of social distance can form an impervious elite network because they face lower transaction costs associated with communication and coordination.

The results indicate that a cooperative equilibrium is feasible only within the elite network. Conversely, agents beyond the threshold level of social distance comprise the nonelite. The size of the elite indicates the exclusivity of the network: the smaller the value of *sN*, the higher the concentration of political power.

2.1. Households

Based on the initial characterization of the economy, we assume that there are two types of households: elite and nonelite. The agents in each group are homogenous and have identical preferences. The household head maximizes family utility by incorporating the welfare of the next generation. He or she also earns the highest income. This argument has an important implication: elite agents (if present) in the household will always be its decision makers, given their significant contribution to household income.

Households maximize their lifetime utility *U*, measured as the weighted sum of future discounted utility flows. Utility is derived through the consumption of good c_t. The utility function assumes the properties of a constant intertemporal elasticity-of-substitution function (CIES) such that the Inada conditions are satisfied.

$$U = \int_0^{\infty} u(c_t) e^{-pt} . dt \tag{1}$$

Each household is endowed with accumulated assets *a* earning a rate of return *r*. All agents are competitive and earn a return equivalent to their marginal product. Each unit of labor supplied is inelastic at the prevalent wage rate, *w*. Elite households have three sources of income: resource rents ($\frac{\sigma \tilde{R}}{sN}$), interest income (*r*) and wages (*w*).

All elite agents are homogenous in terms of their access to political power and therefore receive an equal share of the pool of rents. Nonelite households derive their income from the following sources: wages, interest income and profits from the production sector. The elite decide the optimal level of rents imposed on the economy's natural resource flows through a utility maximization problem with respect to σ_t and c_t. Thus, the main distinction between the utility maximization problem of the elite and private households is the constraint function.

2.2. Technology

The economy has one final production sector that produces a single consumption good, c_t. Firms use three factors of production: labor (L), productive capital (K_t) and natural resource flows (R_t). R_t represents the depletion of the natural resource stock S_0, defined as

$$\dot{R} = S_0 - \int_0^t R_t \tag{2}$$

All markets are competitive, with each earning its marginal product. Here, we borrow from Romer's (1986) endogenous growth model to show how, in the presence of diminishing marginal returns, factor inputs will earn constant returns. We solve for equilibrium in the steady state in the presence of learning-by-investing activities and knowledge spillovers from productive capital across firms.

The firm-level production function is similar to that of Gylfason and Zoega (2006), but adjusted for rent-seeking activities. Rents are imposed on natural resources such that only $(1-\sigma)$ reaches the final output sector. The production function for firm i is:

$$Y_{it} = (qK_{it})^{1-\alpha-\beta}(K_t R_{it})^{\beta}(K_t L_i)^{\alpha} \tag{3}$$

Here, q is the exogenous productivity of capital, ranging from 0 to 1. The total stock of productive capital (K_t) in the economy boosts the productivity of other factors of production. Aggregating the per-capita production function across firms yields the standard Cobb–Douglas constant returns function:

$$y_t = Ak_t^{1-\alpha-\beta}((1-\sigma_t)\tilde{R}_t)^{\beta} \tag{4}$$

The marginal product of each factor of production is given by:

$$MPK = (1-\alpha-\beta)Ak^{-\alpha-\beta}\left((1-\sigma_t)\tilde{R}_t\right)^{\beta} = \frac{(1-\alpha-\beta)y_t}{k_t} = r \tag{5}$$

$$MPR = \beta Ak^{1-\alpha-\beta}((1-\sigma)\tilde{R})^{\beta-1} = \frac{\beta y_t}{\tilde{R}} \tag{6}$$

$$MPL = \alpha y_t \tag{7}$$

σ reduces the share of each input factor and is treated as a leakage in the production process.

2.3. Degree of Rent Expropriation

In this analysis, the rents accruing from natural resource flows give the elite an incentive to engage in corrupt behavior. The question is how institutions make it easier for the elite to expropriate these rents. In ethnically polarized societies, the lack of accountability allows the elite to benefit at the expense of the rest of society (see Leite & Weidmann, 1999).

From a cost perspective, agents with a similar ethnic background will find it more cost-effective to interact with their peers. Sharing cultural and normative values, they face lower transaction costs. Moreover, all members of the elite receive constant marginal benefits in the form of expropriated rents. This argument reflects that of Mehlum et al. (2006), who find that all agents involved in rent seeking share these rents equally among themselves. As a result, corruption becomes profitable among the elite, given the lower marginal costs and higher marginal benefits involved.

2.4. The Cost to the Elite and Concentration of Political Power

Generally, the elite face two different costs: transaction costs and penalty costs. Transaction costs are a priori and determined by inherent characteristics such as ethnicity. Penalty costs materialize if a member of the elite is caught engaging in corruption. Penalty costs are an explicit component of the elite's utility maximization problem, while transaction costs are implicit in determining the size of the elite (see Section 2.7).

2.4.1. Transaction Costs

The concept of social distance is used to quantify transaction costs in this analysis. Sociological studies emphasize the importance of family ties and extended kinship in facilitating interaction between individuals through lower transaction costs. Langlois (1992) classifies these as the cost of (i) persuading, (ii) coordinating and negotiating and (iii) teaching others. In economics, these include the cost of "administering, directing, negotiating and monitoring … joint productive teamwork" as well as the "quality or performance of contractual agreements" in firm behavior theory (Alchian & Woodward, 1988, cited in Langlois, 1992, p. 102).

We apply the concept of transaction costs to a more political-economic setting by categorizing them as either coordination costs or contract enforcement costs. Coordination costs increase with the size of the social network: it is more cost-effective for two independent agents to interact as opposed to two pairs of agents connected through a network. Similarly, contract enforcement costs, which originate from social contract theory, involve an implicit agreement between the members of a society, backed by moral obligations. In the presence of mutual trust, these costs tend to be lower, thereby facilitating interaction between agents with a similar cultural and ethnic identity. Our aim is to show how a corrupt elite are more likely to trust agents within their own ethnic group and collude with them at a high-corruption equilibrium.

An individual's social distance (z_i) from the political center will determine his or her access to and entry into the elite network. We assume that the total population is uniformly distributed against z_i. This is treated as a random variable ranging from 0 to $\bar{Z}$ as individuals cannot choose their ethnic or cultural background. z_i has two threshold values, z_1 and z_2, which determine the size of the elite and nonelite, respectively. For all values of z_i ranging from 0 to z_1, all agents have an equal probability of becoming a member of the elite. The size of the elite is determined endogenously through a cost–benefit analysis (Section 2.5).

The asymmetry in the social distance between different members of society culminates in diverging transaction costs. Only agents closer to the political center (a smaller social distance) can penetrate the elite network, forming a distinct culture that comprises similar beliefs, values and language. This insider advantage results in lower transaction costs (T_i), which are quantified as:

$$T_i = \theta z_i \tag{8}$$

θ is a fixed parameter denoting the magnitude of the transaction costs. Intuitively, it represents the degree of polarization in society, where a higher value of θ corresponds to greater divergence in cultural and normative values across agents. We also assume that members within a network are homogenous and therefore face uniform costs within a group.

2.4.2. Penalty Costs

The second dimension of transaction costs relates specifically to a corrupt elite. These are termed penalty costs: the cost paid by the elite if caught in an act of rent expropriation. Diverging from Leite and Weidmann (1999), we assign a different functional form to these costs. Penalty costs (Q) are taken as a function of a fixed fine ($\bar{Q}$) paid in proportion to the degree of involvement in rent-seeking activities (σ). These costs materialize once the corrupt (elite) agent is caught. Such costs are nonpecuniary, such as the loss of reputation or years of imprisonment. Penalty costs are uniform among the elite in lieu with US federal sentencing guidelines and are defined as:

$$Q(\sigma, \bar{Q}) = \sigma\bar{Q} \qquad (9)$$

The probability (p) of a member of the elite being caught in rent-seeking activities is endogenous and depends on the degree of rent seeking (σ) and monitoring efficiency (m) in the economy. Mathematically, this relationship is expressed as:

$$p(\sigma, m) = \sigma m \qquad (10)$$

In both cases, σ and m, this is an increasing function. A rise in rent-seeking activities will result in higher levels of corruption, thereby increasing the likelihood of being caught, $\frac{dp}{d\sigma}$. Similarly, an efficient monitoring system will increase the chances of detecting corrupt behavior, $\frac{dp}{dm}$. Thus, by modelling monitoring efficiency, we capture indirectly the effect of institutional quality. A more transparent monitoring system is synonymous with more inclusive institutions. In this framework, the probability of getting caught and the degree of rent expropriation are determined jointly, while the monitoring system is exogenous.

Combining equations 9 and 10, total penalty costs are defined as:

$$\textit{Total penalty costs} = \sigma^2\bar{Q}m \qquad (11)$$

Equation 11 embodies the importance of the quality of institutions in determining how well a society functions. Without inclusive institutions to internalize these costs, the latter become ineffective. Our results show that institutional quality – through monitoring efficiency – plays a key role in activating the natural resource curse (see also Mehlum et al., 2006; Leite & Weidmann, 1999).

2.5. The Elite Decision Problem

The elite decision problem has two dimensions. First, agents must decide whether to become members of the elite, based on a cost–benefit analysis. Here, the marginal cost is the transaction cost involved, while the marginal benefit includes the rents accruing from natural resource flows. Since all agents within a group are homogenous, this implies that all members of the elite will be corrupt. The result is a high-corruption (low-growth) equilibrium.

Once the elite decide to become rent seekers, the second tier of the decision concerns the optimal level of rent (σ): this is derived from the elite utility maximization problem. The model is solved using backward induction. The elite maximize their utility subject to an income constraint. The utility function in equation 1 is subject to the following budget constraint in per-capita terms:

$$\dot{y} = w + ra + \frac{(1-p(\sigma,m))\sigma_t \tilde{R}_t}{sN} - p(\sigma,m)\sigma_t \bar{Q} - c_t \tag{12}$$

The elite derive their income from three sources: wages (*w*), the return on assets (*ra*) and natural resource rents $\left(\frac{(1-p(\sigma,m))\sigma_t \tilde{R}_t}{sN}\right)$. Note that natural resource rents are secured only with the probability of not getting caught $((1-p(\sigma,m))$. If a rent seeker is caught with a probability of p, the expropriated rents are dissipated as monitoring costs and treated as efficiency losses. In addition, the elite have to pay a penalty cost of $p(\sigma,m)\sigma_t \bar{Q}$. Elite households spend c_t on consumption over their lifetime.

We set up a Hamiltonian problem to solve the utility maximization problem, using the degree of rent expropriation (σ_t) and consumption (c_t) as the choice variables in equation 13. y_t and λ_t are taken as the state and co-state variables, respectively.

$$H = \int_0^\infty u(c_t)e^{-pt}.dt + \lambda_t(w + ra + \frac{\big(1-p(\sigma,m)\big)\sigma_t \tilde{R}_t}{sN}$$
$$-p(\sigma,m)\sigma_t \bar{Q} - c_t) \tag{13}$$

The first-order conditions yield the following results:

$$\frac{dH}{dc_t} = u'(c_t)e^{-pt}.dt = \lambda_t \tag{14}$$

$$\frac{dH}{d\sigma_t} = \frac{\tilde{R}_t}{sN} - \frac{2\sigma_t m\tilde{R}_t}{sN} - 2\sigma_t m\bar{Q} = 0 \tag{15}$$

$$\frac{dH}{dy_t} = \dot{\lambda} = -\lambda_t r \tag{16}$$

Solving equation 15 for $\sigma_t(R_t)$ and $1 - (\sigma_t(R_t)$:

$$\sigma_t(\tilde{R}_t) = \frac{\tilde{R}_t}{2m\tilde{R}_t + 2m\bar{Q}sN} \tag{17}$$

$$1 - \sigma_t(\tilde{R}_t) = \frac{2m\tilde{R}_t + 2m\bar{Q}sN - \tilde{R}_t}{2m\tilde{R}_t + 2m\bar{Q}sN} \tag{18}$$

$\sigma_t(R_t)$ maximizes the utility of the elite household subject to the income constraint. σ_t decreases with m, implying that higher levels of monitoring efficiency deter rent-seeking activities. Relating this discussion to the quality of institutions, we can see how inclusive institutions are likely to mitigate corruption by creating a more transparent accountability system. In a similar vein, penalty costs tend to decrease rent expropriation activities. Since these costs are internalized in the elite decision-making process ex-ante, increasing the magnitude of the fine will reduce the degree of corruption.

Finally, an increase in the size of the elite (sN) – synonymous with weaker retention of power within the elite network – will reduce rent expropriation. As the size of the elite grows, they become less exclusive, making it difficult to collude in a high-corruption equilibrium. These results support Shleifer and Vishny (1993) and Leite and Weidmann (1999), who find that the concentration of political power is associated with efficiency losses from corruption.

2.6. Final Production Sector

After normalizing prices, the firm's profit function is:

$$\pi_{it} = (qK_{it})^{1-\alpha-\beta}(K_t(1-\sigma_t)R_{it})^{\beta}(K_tL_i)^{\alpha} - w - I_{it} \tag{19}$$

Firms maximize profits by investing I_{it} in productive capital K_{it} and natural resource flows R_{it} from the stock of natural resources S_0.[3] We use a present-value Hamiltonian problem to solve the firm's profit maximization problem:[4]

$$H = \left[(qK_{it})^{1-\alpha-\beta}\big((1-\sigma_t)K_tR_{it}\big)^{\beta}(K_tL_i)^{\alpha} - w - I_{it}\right]e^{-rt} + \mu_0(R_t)$$

$$-\mu_1(R_t) + \lambda_1(I_{it} - \delta K_{it}) \tag{20}$$

Investment and natural resource flows are used as optimizing variables subject to the capital stock constraint and natural resource constraint. The first-order condition yields the following results:

$$\frac{dH}{dI_{it}} = (-1)e^{-rt} + \lambda_1 = 0 \tag{21}$$

$$\dot{\lambda} = -\frac{dH}{dK_{it}} = -(1-\alpha-\beta)q^{1-\alpha-\beta}(K_{it})^{-\alpha-\beta}\big((1-\sigma_t)K_tR_{it}\big)^{\beta}$$

$$(K_tL_i)^{\alpha} + \lambda\delta \tag{22}$$

$$\frac{dH}{dR_{it}} = \beta - 1(qK_{it})^{1-\alpha-\beta}(K_t)^{\beta}(1-\sigma_t)^{\beta}{R_{it}}^{\beta-1}(K_tL_i)^{\alpha} = 0 \tag{23}$$

$$\dot{\mu} = \frac{dH}{dS_0} = 0 \tag{24}$$

Based on the profit maximization problem, we calculate the steady-state values of σ^* and $\tilde{R}^*$ in Section 3.

2.7. The Size of the Elite

The final stage of the model involves determining the size of the elite. Using the optimal values of σ^* and R^* in the steady state, the value of *sN* is

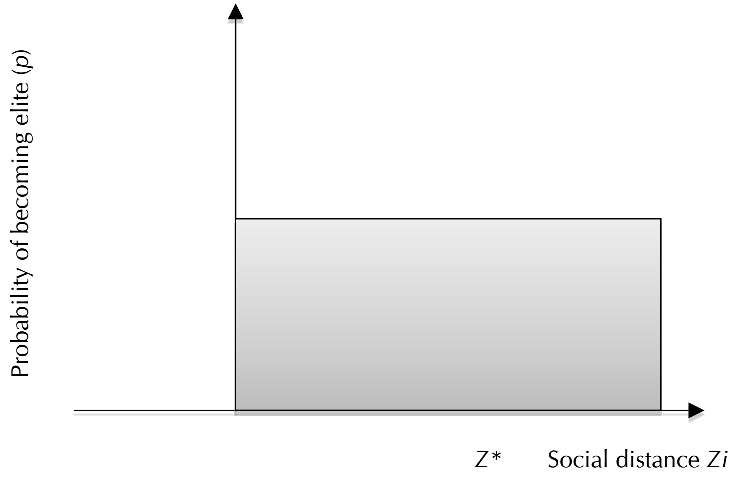
[3] At the firm level, firms maximize their profits, taking into account only K_i: the spillover effect of the aggregate level of capital stock is taken as given. Thus, despite the diminishing marginal returns of the production function, knowledge spillovers lead to constant returns to scale at the aggregate level.

[4] $\mu_0(R_t)$ is the nonnegative constraint to natural resource flows. We use the Kuhn–Tucker conditions of complementary slackness to solve the model such that $\mu_0(R_t) = 0$. Thus, the natural resource constraint $-\mu_1(R_t)$ is reduced to $-\mu_1(R_t)$ (see Dasgupta & Heal, 1974).

determined endogenously by weighing the marginal benefit against the marginal cost. At this point, we incorporate transaction costs as the only costs.[5] The marginal benefit is the share of natural resource flows per member of the elite. As Figure 1 shows, the elite population follows a normal distribution, with social distance as the random variable:

$$\frac{1}{\bar{Z}} * z_1 = sN \quad (25)$$

Figure 1: Uniform distribution of the elite with respect to social distance



Based on the transaction costs defined in equation 8, we combine equations 8 and 25 and express transaction costs[6] in terms of *sN*. Normalizing $\bar{Z}$ results in the equation below:

$$T_1 = sN\theta \quad (26)$$

[5] Penalty costs are incorporated in the elite decision-making process through σ^* and R*.
[6] We take social distance as an exogenous variable predetermined by birth. It can be acquired in the sense that individuals' beliefs and values are shaped by their interaction with other members of that ethnic group. The caveat to this assumption is that we do not account for inter-ethnic interaction, which the model deems costly and thus not feasible for the individual. This implies that external ethnic interaction will never materialize.

Equating the expected net marginal benefit with the marginal cost at z_1 yields:

$$sN\theta = \frac{(1-\sigma m)\sigma^{*}\tilde{R}^{*}}{sN} - (\sigma^{2} m\bar{Q}) \quad (27)$$

See Appendix 1 for detailed calculations.

3. High-Corruption (Low-Growth) Equilibrium

This section presents the results of the model in a steady-state equilibrium, based on which we determine the impact of rent seeking on growth.

3.1. Rent Expropriation

The results of the elite utility maximization problem determine how the degree of rent expropriation is affected by natural resource flows, penalty costs and the size of the elite[7] (see Appendix 2):

$$\sigma_t(R_t) = \frac{R_t}{(2mR_t + 2m\bar{Q}sN)} \quad (28)$$

$$\frac{\dot{c}}{c} = \frac{r-\rho}{\varphi} \quad (29)$$

An increase in natural resource flows will increase the expropriated rents imposed by the elite. Natural resource flows increase the returns to the rent seeker by increasing the pool of rents that is subsequently shared equally among the elite. Note that natural resource flows can be directed toward productive as well as rent-seeking activities. The extraction of natural resources is not the problem per se, but the allocation of resource flows is a critical factor in triggering the natural resource curse.

We find a nonlinear relationship between natural resource flows and rent-seeking activities. There are two forces at play here. At lower levels of extraction, the returns accruing to the elite exceed the penalty costs imposed, thereby providing greater incentive for corrupt behavior. Conversely, at higher levels of natural resource flows, the negative feedback effect of penalty costs overrides the return on corruption. This weakens the positive relationship between σ_t and R_t.

[7] The result for the optimal consumption path reflects the standard findings of the CIES utility function. Note that the optimal consumption path of the elite and the nonelite is uniform.

The quality of institutions, quantified by monitoring efficiency, has a negative relationship with the degree of rent expropriation by the elite. This result supports earlier studies such as Acemoglu, Robinson and Verdier (2003). Higher levels of monitoring efficiency imply that the actions of the elite are subject to greater accountability, making it more difficult to divert rents from the economy's productive sector. Synonymously, inclusive institutions deter collusion among the elite by making corruption (the redistribution of wealth from the nonelite to the elite) costlier.

sN quantifies the concentration of political power, where a smaller value of *sN* corresponds to the greater retention of power by fewer agents. Unsurprisingly, the concentration of political power facilitates rent seeking by making it easier for bureaucrats to expropriate rents. Conversely, a higher value of *sN* decreases the degree of corruption by diluting the insider advantage of the elite. An increase in the size of the elite network implies that the cost of coordination and communication within this group will also rise, thereby making cooperation among the elite less feasible.

These findings corroborate the results of political economy models that show how the concentration of political power allows the elite to build coalitions in the presence of resource rents. This study shows how ethnic polarization facilitates the concentration of political power when the size of the elite is determined.

Finally, penalty costs ($\bar{Q}$) internalize the cost of corruption by reducing the expropriation of rents by the elite (yielding a negative relationship). Increasing the magnitude of the fine will counteract the actions of a corrupt elite.

3.2. Steady-State Solution

The results of the firm profit maximization problem in the steady state can be reduced to the following time derivatives:

$$\frac{\dot{\sigma}}{1-\sigma} = \frac{\dot{R}}{R} = -r \tag{30}$$

Linking the firm and the elite utility maximization problem, we can determine the optimal level of rent seeking and natural resource extraction using the condition above[8] (see Appendix 3).

$$R^* = \frac{\left(\sqrt{1-2m}-(1-2m)\right)\bar{Q}sN}{(1-2m)} \quad (31)$$

$$\sigma^* = \frac{\sqrt{1-2m}-(1-2m)}{2m\sqrt{1-2m}} \quad (32)$$

In equilibrium, we find the optimal level of natural resource extraction by profit maximizing firms. The decision to extract natural resources is made after firms have accounted for rent expropriation and its constraints. R^* can be interpreted as the resources diverted toward productive activities. The results indicate a positive relationship between penalty costs and natural resource extraction. Similarly, a lower concentration of political power translates into greater allocative efficiency because more resources are diverted toward the growth-inducing sector.

The optimal level of rent expropriation, σ^*, is solely a function of monitoring efficiency. The literature emphasizes the importance of the quality of institutions in activating the natural resource curse. Similarly, our results echo the importance of the role of institutions in determining the growth trajectory of an economy.

A nonmonotonic relationship exists between m and σ^*. At lower levels of m, σ^* is an increasing function of m. In other words, at lower levels of monitoring efficiency, corruption through rent seeking is less costly for the elite. However, at higher levels of monitoring efficiency, the elite are held more accountable for their actions, which hinders the diversion of productive resources toward rent seeking. One of the most significant findings of this study is that the elite can expropriate a greater share of natural resource flows only in the presence of extractive institutions.

3.3. The Elite Network

The size of the elite is determined endogenously through a marginal cost and expected marginal benefit analysis. Once the degree of rent

[8] The optimal solutions are applicable for all values of $0 < m < ½$. The results assume that any economy's monitoring capability is not perfectly efficient. This implies that the model determines outcomes only for economies endowed with low-quality institutions.

expropriation and optimal natural resource flows have been determined, the size of the elite can be calculated. Using equation 28 and plugging in the optimal values of natural resource flows and the degree of rent expropriation, we obtain the size of the elite in the steady state:

$$sN = \frac{\left(\sqrt{1-2m}-1+2m\right)\bar{Q}\left(1-\sqrt{1-2m}\right)}{4m(1-2m)\theta} \tag{33}$$

We can now address the study's main aim, with ethnic polarization at the center of analysis. θ reflects the divergence in cultural and normative beliefs of the elite and nonelite. In other words, it denotes how far apart they stand on the social distance scale. The size of the elite decreases in θ, showing how ethnic polarization facilitates the concentration of political power.

In polarized societies, political parties rely on coalitions built within their own ethnic group. Only the dominant ethnic group can secure political power at the center. Our results support this intuitive argument: only agents closer to the political center can form an impenetrable elite network, while barriers to entry arise from the difference in transaction costs between the two groups.

As before, the magnitude of penalty costs reduces the concentration of political power. For all values of *m* less than ½, the elite are aware of the repercussions of engaging in corruption, demonstrating the negative relationship between penalty costs and the concentration of power. Our results differ from that of Leite and Weidmann (1999), who find that the magnitude of penalty costs becomes ineffective in the presence of poor-quality institutions. In our model, this magnitude internalizes the cost of corruption at both stages of decision making, thereby reducing the concentration of political power.

3.4. Threshold-Value Natural Resource Extraction

The threshold value of natural resource flows that will ensure a high-corruption equilibrium can be determined through a marginal cost–benefit analysis. Beyond a certain value of *R*, the expected marginal benefit of rent seeking exceeds the marginal cost. The results are presented in the equation below:

$$\underline{\tilde{R}}_t = \frac{sN(\sigma_t^2 m\bar{Q}+sN\theta)}{\sigma_t(1-\sigma m)} \tag{34}$$

Plugging in the optimal values of σ^* and $sN(\theta)$, we determine the threshold value of natural resource flows in the steady state for ensuring a high-corruption equilibrium (see Appendix 4):

$$\underline{\tilde{R}}_t = \frac{A\bar{Q}(1-\sqrt{1-2m})(1-\sqrt{1-2m}-A^2)}{8m^2(1-2m)^2\theta} \tag{35}$$

where A is defined as $\sqrt{1-2m}-(1-2m)$.

For values of $\tilde{R}_t$ greater than $\underline{R}_t$, the elite have an incentive to become corrupt because the associated net benefit is higher than the cost. Conversely, for all values of $\tilde{R}_t$ below $\underline{R}_t$, the marginal cost (the transaction and penalty costs) drive down the expected net benefit of rent expropriation. The higher the threshold value of $\underline{R}_t$, the greater the range of natural resource flows that can sustain a low-corruption equilibrium. Interestingly, the dilution of political power corresponding to a larger value of *sN* increases the value of $\underline{R}_t$. By weakening the elite network, the economy can move to a high-growth trajectory even in the presence of greater natural resource extraction and vice versa.

In equation 35, the role of ethnic polarization through θ is key to lowering the threshold value of $\underline{R}_t$. By increasing the transaction cost of political interaction, θ enables the formation of a tightly knit elite network. Thus, the degree of polarization in the presence of low-quality institutions determines the growth trajectory of an economy rich in natural resources. As before, penalty costs have a similar effect by making the economy less vulnerable to the natural resource curse.

The study's proposition is that $\nabla m \leq \frac{1}{2} \Rightarrow \tilde{R}_t > \underline{\tilde{R}}_t$: all members of the elite are corrupt, with a high concentration of political power, such that the economy converges toward a low-growth equilibrium (see Appendix 4 for the mathematical proof).

We impose a restriction on *m* such that *m* ranges from 0 to ½, as a result of which the economy is always trapped in a high-corruption equilibrium in the presence of extractive institutions. The quality of institutions plays a vital role through two channels in this model. First, grabber-friendly institutions make rent expropriation easier because the elite are not held accountable. Low levels of monitoring efficiency increase the marginal benefit of rent seeking by lowering the probability of getting caught. Second, on the cost side, extractive institutions – quantified by a smaller value of m – mitigate the magnitude of the penalty cost in the elite's budget constraint. Thus, poor institutional quality increases the net returns accruing to a corrupt elite such that the optimal extraction of natural resource flows exceeds $\underline{\tilde{R}_t}$.

3.5. Consumption and Optimal Stock of Capital

This section examines how our results affect the growth dynamics of the economy in a high-corruption equilibrium. The optimal level of natural resource extraction ($\tilde{R}^*$) and the degree of rent expropriation (σ^*) for a given elite network $(sN)^*$ determine the steady-state level of capital stock (K^*). In a competitive equilibrium framework, the marginal product of capital is determined by combining equations 21 and 22 from the firm profit maximization problem:

$$MPK = r + \delta \tag{36}$$

Plugging in the optimal values of $\tilde{R}^*$ and $(1-\sigma^*)$, the optimal level of capital stock in the steady state can be defined as:

$$K^* = \left(\frac{A(1-\alpha-\beta)}{r+\delta} \left(\frac{\sqrt{1-2m}(1-m)-(1-2m)}{\sqrt{1-2m}} QsN(\theta, Q) \right)^{\beta} \right)^{\frac{1}{\alpha+\beta}} \tag{37}$$

Similarly, we use equation 29 to determine the optimal consumption path in the steady state (see also Appendix 5):

$$\frac{\dot{c}}{c} = \frac{MPK^* - \delta - \rho}{\varphi} = 0 \tag{38}$$

The natural resource curse comes into play in a high-corruption equilibrium: the higher the share of natural resources in the national output (denoted by β), the lower the steady-state level of capital accumulation. Again, it is not the presence of natural resources per se that is the problem, but rather the rent expropriation activities associated with these flows. Our argument here is that rent seeking becomes easier in the presence of concentrated political power.

These results support the hypothesis that a closely knit elite network, quantified by a smaller value of sN, reduces the optimal stock of capital. This finding is similar to that of Mehlum et al. (2006), who show that, in the presence of natural resource flows, productive resources may be diverted toward rent-seeking activities, thereby lowering investment in capital.

Incorporating ethnic polarization adds another dimension to the transmission mechanism. Our results show that it is ultimately the degree of ethnic polarization, θ, that triggers the natural resource curse: θ has a negative impact on the optimal level of capital stock through sN. In highly polarized societies, therefore, the formation of an elite network concentrates power among fewer agents. This shift in the nexus of political power from the nonelite to the elite provides incentive for rent seeking, in turn lowering investment in growth-inducing activities. This leaves the economy trapped in a low-growth equilibrium and perpetuates the vicious cycle between resource rents and extractive institutions.

3.6. Income Redistribution Impact of Natural Resource Flows

In our model, the elite have three sources of income: wages, returns on savings and the net rents received from natural resource flows. Alternatively, the nonelite receive wages, returns on savings and a share of the nonexpropriated rents from natural resource flows. The income comparison between the corrupt elite and the nonelite in equilibrium can be defined as:

$$w + ra + \frac{(1-\sigma m)\sigma^* R^*}{sN} - \sigma^2 mQ - c_t > w + ra + \frac{(1-\sigma^* m)(1-\sigma^*)R^*}{(1-s)N} \qquad (39)$$

Plugging in the optimal values of R^* and σ^*, we can determine the parameters that accentuate the difference in income levels (see also Appendix 6):

$$2msN - \sqrt{1-2m} + (1-2m) - \sqrt{1-2m}\frac{s}{1-s} > 0 \qquad (40)$$

The comparative statistics with respect to the size of the elite yield a negative function, indicating that the concentration of political power redistributes income from the nonelite to the elite. This finding reflects the same idea discussed earlier: that rent-seeking activities divert resources from productive to nonproductive sectors. Again, ethnic polarization is at the center of this power play. In the presence of extractive institutions, it not only sets the economy on a trajectory of low growth, but also increases income inequality.

The collective action problem is a prominent feature of ethnically polarized societies and is worse under majoritarian rule than under a proportional electoral system. Our model assumes the latter, based on Downs' (1957) median voter theorem. In a majoritarian system, the dominant party makes decisions that may be unfavorable to the losing party. Our findings predict a similar outcome: the elite tend to perpetuate a vicious cycle of excluding the nonelite from political decision making and other policy variables. Therefore, inequality can have far-reaching effects not just on growth, but also on the development trajectory of the economy.

4. Conclusion

The irregularities in the empirical literature on the natural resource curse make it necessary to develop a unifying framework to address the issue. This study uses a dynamic, infinite time horizon model to build a theoretical framework that posits a nonmonotonic relationship between natural resource extraction and growth. The model determines a threshold level of extraction beyond which economies with kleptocratic institutions become trapped in a low-growth equilibrium.

While Mehlum et al. (2006) stress the importance of institutional quality in explaining the natural resource curse, it is important to disassemble the impact of institutional quality on the growth trajectory of an economy. Our model takes institutional quality (quantified by monitoring efficiency) as exogenous, but also accounts for circumstances in which the former might deteriorate. We argue that ethnic polarization connects the missing dots in the literature. The study's empirical findings corroborate the thesis that, in the absence of political competition, the incumbent government will divert resources from entrepreneurial activities.

The model's findings predict a similar fate for polarized societies where the cost of communicating with outsiders is high enough to deter any form of interaction with them. Ethnic polarization concentrates political power through the formation of a closely knit elite network. This network gives its members incentive to collude with each other in corrupt activities by increasing the net share of resource flows per member of the elite.

The implications of a strong elite network extend to the degree of rent expropriation, where concentrated political power is associated with greater expropriation. A feedback mechanism translates greater expropriation into the diversion of productive resources away from the final output sector. Firms tailor their actions accordingly and invest less in productive capital.

These findings are in accordance with Torvik (2002), who predicts that productive capital will bear the brunt of rent-seeking activities. If productive capital is the engine of growth, curbing investment will drive down the aggregate income of an economy. We define productive capital in a wider sense, referring not just to physical capital, but also to all learning-by-doing activities. This makes it possible to address the conditionality of the natural resource curse. The study also compares the income levels of the elite and the nonelite, finding that the concentration of political power through ethnic polarization worsens income inequality across groups in the equilibrium state.

One caveat is that these results can be generalized only across economies with a weak accountability system. That said, the assumption is not restrictive because it appeals to the economic rationale that the quality of institutions helps determine the fate of an economy.

Our results are important from a policy perspective in two ways. First, economies endowed with kleptocratic institutions fall prey to the natural resource curse because such institutions increase the net marginal benefit and decrease the cost of corruption. High-quality institutions can help overcome the natural resource curse by creating a more transparent bureaucratic structure. Although the black box of institutional quality is not addressed in this model, it is logical to assume that greater investment in education and health can improve institutional quality.

Second, ethnic polarization enables corruption by creating a wedge in the form of social distance between members of society. Socially inclusive policies can help bridge the gap between different ethnicities. Education policies, for example, should be designed to inculcate a sense of belonging and equality among all members of a society irrespective of their ethnic background.

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Appendix 1: The size of the elite

The size of the elite is calculated as follows. Equating the marginal cost with the expected net benefit and plugging in the optimal values of natural resource flows and rent expropriation in the steady state, we obtain:

$$\frac{(1-\sigma^{*}m)\sigma^{*}R}{sN} - \sigma^{2}mQ = \theta sN$$

Using $A = \sqrt{1-2m} - (1-2m)$

$$\frac{AQsN}{2(1-2m)} - \frac{A^{2}}{4m^{2}(1-2m)}mQ = \theta sN$$

$$\frac{AQ}{2(1-2m)} - \frac{A^{2}Q}{4m(1-2m)} = \theta sN$$

$$sN = \frac{\left(\sqrt{1-2m} - 1 + 2m\right)\bar{Q}\left(1-\sqrt{1-2m}\right)}{4m(1-2m)\theta}$$

Appendix 2: Elite optimal consumption and rent seeking

We solve for the optimal consumption and rent-seeking path of the elite using the utility maximization problem below:

$$H = \int_0^\infty u(c_t)e^{-\rho t}.dt + \lambda_t(w + ra + \frac{(1 - p(\sigma, m))\sigma_t \tilde{R}_t}{sN} - p(\sigma, m)\sigma_t \bar{Q} - c_t)$$

The first-order conditions are:

$$\frac{dH}{dc_t} = u'(c_t)e^{-\rho t}.dt = \lambda_t$$

$$\frac{dH}{d\sigma_t} = \frac{\tilde{R}_t}{sN} - \frac{2\sigma_t m \tilde{R}_t}{sN} - 2\sigma_t m \bar{Q} = 0$$

$$\frac{dH}{dy_t} = \dot{\lambda} = -\lambda_t r$$

Combining $\frac{dH}{d\sigma_t}$ and $\frac{dH}{dy_t}$ gives us:

$$\tilde{R}_t - 2\sigma m \tilde{R}_t - 2\sigma m \bar{Q} sN = 0$$

$$\sigma_t = \frac{\tilde{R}_t}{2m\tilde{R}_t + 2m\bar{Q}sN}$$

Combining $\frac{dH}{dc_t}$ and $\frac{dH}{dy_t}$ and using a CIES function, we obtain:

$$\frac{u''(c_t)}{u'(c_t)} = -\frac{\varphi}{c}$$

$$-\varphi\frac{\dot{c}}{c} = -r + \rho$$

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\varphi}$$

We construct the expression below for $1 - \sigma_t$:

$$\frac{2m\tilde{R}_t + 2m\bar{Q}sN - \tilde{R}_t}{2m\tilde{R}_t + 2m\bar{Q}sN}$$

Thus, $\frac{\dot{\sigma}}{1-\sigma}$ can be expressed as:

$$\log(1-\sigma_t) = \log\left(2m\tilde{R}_t - \tilde{R}_t + 2m\bar{Q}sN\right) - \log(2m\tilde{R}_t + 2m\bar{Q}sN)$$

Incorporating time derivatives, we obtain:

$$-\frac{\dot{\sigma}}{1-\sigma} = \frac{1}{2m\tilde{R}_t - \tilde{R}_t + 2m\bar{Q}sN}(2m-1)\dot{\tilde{R}} - \frac{1}{2m\tilde{R}_t + 2m\bar{Q}sN}(2m\dot{\tilde{R}})$$

Appendix 3: The profit maximization problem

The results of the profit maximization problem are expressed as:

$$H = \left[(qK_{it})^{1-\alpha-\beta}\big((1-\sigma_t)K_tR_{it}\big)^{\beta}(K_tL_i)^{\alpha} - w - I_{it}\right]e^{-rt} + \mu_0(R_t) - \mu_1(R_t) + \lambda_1(I_{it} - \delta K_{it})$$

The first-order conditions are:

$$\frac{dH}{dI_{it}} = (-1)e^{-rt} + \lambda_1 = 0$$

$$\dot{\lambda} = -\frac{dH}{dK_{it}} = -(1-\alpha-\beta)q^{1-\alpha-\beta}(K_{it})^{-\alpha-\beta}\big((1-\sigma_t)K_tR_{it}\big)^{\beta}(K_tL_i)^{\alpha} + \lambda\delta$$

$$\frac{dH}{dR_{it}} = \beta - 1(qK_{it})^{1-\alpha-\beta}(K_t)^{\beta}(1-\sigma_t)^{\beta}R_{it}{}^{\beta-1}(K_tL_i)^{\alpha} = 0$$

$$\dot{\mu} = -\frac{dH}{dS_0} = 0$$

Combining $\frac{dH}{dI_{it}}$ and $\dot{\lambda} = -\frac{dH}{dK_{it}}$ and incorporating time derivatives, we obtain:

$$\log(1-\alpha-\beta) + (1-\alpha-\beta)\log q + \beta\log(1-\sigma_t) + \beta\log R_t + \alpha\log L = \log(r+\delta)$$

$$-\beta\frac{\dot{\sigma}}{1-\sigma} = -\beta\frac{\dot{R}}{R}$$

Combining $\frac{dH}{dR_{it}}$ and $\dot{\mu} = -\frac{dH}{dS_0}$ yields the following derivatives:

$$\frac{\dot{K}}{K} = \beta\frac{\dot{\sigma}}{1-\sigma} + r + (1-\beta)\frac{\dot{R}}{R}$$

Keeping $\frac{\dot{K}}{K} = 0$ in the steady state, we obtain the following dynamic equation:

$$\frac{\dot{\sigma}}{1-\sigma} = \frac{\dot{\tilde{R}}}{R} = -r$$

Using the steady-state condition of the firm's profit maximization problem to solve for σ^* and $\tilde{R}^*$ yields

$$\frac{1}{R} = \frac{2m}{2mR + 2m\bar{Q}sN} - \frac{2m-1}{2mR + 2m\bar{Q}sN - R}$$

$$\frac{1}{R} = \frac{2m\bar{Q}sN}{(2mR + 2m\bar{Q}sN)(2mR - R + 2m\bar{Q}sN)}$$

Using $B = m\bar{Q}sN$, the expression is simplified to:

$$(4m^2 - 2m)\tilde{R}^2 + (4mB - 2B)\tilde{R} + B^2 = 0$$

This generates two solutions:

$$R = \frac{B - 2mB \pm \sqrt{1-2m}}{2m(2m-1)} \neq feasible < 0$$

$$\tilde{R}^* = \frac{-B + 2mB + B\sqrt{1-2m}}{2m(1-2m)} = \frac{\left(2m + \sqrt{1-2m} - 1\right)\bar{Q}sN}{2m(1-2m)} > 0$$

We impose a restriction on *m* such that *m* ranges from 0 to ½.

Finding the optimal value of σ^* using $\tilde{R}^*$ yields

$$\sigma^* = \frac{\sqrt{1-2m} - (1-2m)}{2m\sqrt{1-2m}}$$

Appendix 4: The threshold value of R

We use a marginal benefit–cost analysis to determine the threshold level of R, which ensures a high-corruption equilibrium such that

Net marginal benefit > marginal cost

$$\frac{(1-\sigma^* m)\sigma^* R}{sN} - \sigma^2 mQ = \theta sN$$

Plugging in the optimal value of σ^* yields

$$\underline{R} = 2\left[(sN)^2\theta + sN\frac{(\sqrt{1-2m}-(1-2m))^2\overline{Q}}{4m(1-2m)}\right]$$

Comparing $\tilde{R}^*$ and $\underline{\tilde{R}}$ in a steady-state equilibrium, we find that

$$\tilde{R}^* \geq \underline{\tilde{R}}$$

$$\frac{\left(\sqrt{1-2m}-(1-2m)\right)2\overline{Q}sN}{(1-2m)} \geq 2sN\left(sN\theta + \frac{(\sqrt{1-2m}-(1-2m))^2\overline{Q}}{4m(1-2m)}\right)$$

Using $A = \sqrt{1-2m}-(1-2m)$,

$$2m\overline{Q}A > sN\theta 4m(1-2m) + A^2\overline{Q}$$

Plugging in the value of *sN* gives us:

$$AQ > \left(1-\sqrt{1-2m}-A^2\right)$$

$$\overline{Q}\sqrt{1-2m} - \overline{Q} + 2m\overline{Q} > 1-\sqrt{1-2m}-(\sqrt{1-2m}-(1-2m))^2$$

Since the left-hand side is always greater than the right-hand side, with $Q > 1$ and $0 < m < ½$,

$$\tilde{R}^* > \underline{\tilde{R}}$$

Appendix 5: The optimal level of capital stock and dynamic consumption path

Using $MPK = r + \delta$

$$K^{\alpha+\beta} = \frac{(1-\alpha-\beta)AR^{*\beta}(1-\sigma)^{\beta}}{r+\delta}$$

$$K^{\alpha+\beta} = \frac{(1-\alpha-\beta)A}{r+\delta}\left(\frac{(\sqrt{1-2m}-(1-2m))(1-\sqrt{1-2m})}{2m\sqrt{1-2m}}QsN\right)$$

$$K^{*} = \left(\frac{(1-\alpha-\beta)A}{r+\delta}\right)^{\frac{1}{\alpha+\beta}}\left(\frac{\sqrt{1-2m}(1-m)-1+2m}{2m\sqrt{1-2m}}QsN\right)^{\frac{\beta}{\alpha+\beta}}$$

Similarly, we plug in the optimal values of K^*, $(1-\sigma^*)$ and R^* to find the optimal path of consumption:

$$\frac{\dot{c}}{c} = 0$$

Appendix 6: Comparative income levels for the elite and nonelite

We calculate the comparative income levels of the elite and nonelite as follows:

$$w + ra + \frac{(1-\sigma m)\sigma^* R^*}{sN} - \sigma^2 mQ - c_t > w + ra + \frac{(1-\sigma^* m)(1-\sigma^*)R^*}{(1-s)N}$$

$$\frac{(1-\sigma m)\sigma^* R^*}{sN} - \sigma^2 mQ > \frac{(1-\sigma^* m)(1-\sigma^*)R^*}{(1-s)N}$$

Solving for the left-hand side:

$$\frac{\sqrt{1-2m} + (1-2m)}{2\sqrt{1-2m}} \frac{\sqrt{1-2m} - (1-2m)}{2m\sqrt{1-2m}} \frac{R}{sN} - \frac{\left(\sqrt{1-2m} - (1-2m)\right)^2}{4m^2(1-2m)}$$

$$\frac{\left(\sqrt{1-2m}-(1-2m)\right)\bar{Q}\left(2msN-\sqrt{1-2m}-(1-2m)\right)}{4m^2(1-2m)} = \text{LHS}$$

Solving for the right-hand side:

$$\frac{(1-2m) - \sqrt{1-2m}(1-2m)}{2m\sqrt{1-2m}} \frac{R}{(1-s)N}$$

$$\frac{\left(\sqrt{1-2m} - (1-2m)\right)\bar{Q}s\sqrt{1-2m}}{2(1-2m)(1-s)}$$

Comparing the net return on belonging to the elite versus the nonelite,

$$\frac{\left(\sqrt{1-2m} - (1-2m)\right)\bar{Q}\left(2msN - \sqrt{1-2m} - (1-2m)\right)}{4m^2(1-2m)} > \frac{\left(\sqrt{1-2m} - (1-2m)\right)\bar{Q}s\sqrt{1-2m}}{2(1-2m)(1-s)}$$

This equation can be reduced to

$$2msN - \sqrt{1-2m} + (1-2m) - \sqrt{1-2m}\frac{s}{1-s} > 0$$

The comparative statistics are as follows:

$$\frac{\partial(ye - y_{ne})}{\partial s} = \frac{2mN(1-s)^2 - \sqrt{1-2m}}{(1-s)^2} < 0$$

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Lahore School of Economics
Intersection Main Boulevard Phase VI, DHA and Burki Road
Burki Lahore 53200, Pakistan