Organizational development, foreign technology and economic growth: long-run equilibrium, transitional dynamics and closed-form solutions

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Abstract

We study the dynamics of long run growth in a two sector economy with externalities where organizational development affects physical capital and foreign technology accumulation. The objective of this thesis is to show that the growth model introduced by Lucas which was further developed by Sala-I-Martin and Mulligand, Santos and Caballe , Benhabib and Perli and eventually by Boucekkine and Ruiz-Tamarit, has two compelling properties. Firstly, if the share of physical capital is greater than the externality parameter of foreign technology, then there will be a unique transitional equilibrium path. Secondly, if the share of physical capital is less than the externality parameter in the production function of foreign technology, then the system will lead to multiple steady-states equilibrium i.e, several transitional paths. Furthermore, we derive the closed-form solutions where the elasticity of output with respect to physical capital is equal to the inverse of intertemporal elasticity of substitution, for all the variables in the model to derive the transitional conditions.

DECLARATION

I hereby declare that this thesis is my own work. It is being submitted to Lahore School of Economics for the completion of the Degree of M.Phil Economics. It has not been submitted before for any degree or examination at any other institute.

Contents

INTRODUCTION

Why do some countries grow at a much faster rate than other countries? What causes developing countries to diverge from developed countries in terms of growth rates? The search for these answers has been the focal point of a considerable number of research. Many economists considered output per capita to be associated with the levels of human capital, technology and physical capital in an economy. The idea is that human capital equips people with the kind of knowledge and skills that allows them to increase their level of productivity and also complement technological adaptation allowing the economy to move closer to the technology frontier at a faster pace than other countries. Physical capital accumulation provides labor with access to capital in order to operate. Where such factors play an critical role, they act as proximate causes of economic growth. What makes these factors vary across countries plays a crucial role in examining growth rate differences.

Organizational development is recognized to be the one of the main underlying cause of economic growth. There are significant literature which is based on the development of organizations and economic growth. Organizational development can be defined as the efficient use of resources to improve the productivity and quality of the workplace. It involves the practice of executing organizational change. According to the Beckhard (1969), organizational development can be defined as:

"Organization Development is an effort planned, organization-wide, and managed from the top, to increase organization effectiveness and health through planned interventions in the organization's

'processes,' using behavioral-science knowledge."

Organizations develop for many reasons. Although as, Pfeffer and Salancik (1978), indicates that it is usually hard to acquire the incentive to growth after the fact. According to Tiryakioglu (2006), there exist a positive relationship between organizational development in the form of R&D and economic growth. One of the aim of organizational development is to identify the area in workplace where some change is needed. For this the managers of the company should analyze each need and make some suitable adjustments into a management plan. The study in India by Bloom et al., (2013), ran a field experiment in which they provided free management practice to randomly chosen treatment plants and compare them with the control plants. They saw that these practices helped to improve management practices which resulted on improvement in productivity of 17 percent. The economics of quality improvement have been designed by different industrial organization economists. With the better managing and planning, the quality of the product can also be improved (i.e, quality upgrading) which can help the organization to develop. Beginning with Loury (1979) and Lee and Wilde (1980), much work has been dedicated to understanding the incentives that firms have to introduce new and improved products.

Organizational development can help the economy to grow through different factors including the human and physical capital, practical arrangements, introducing an organizational culture, investments and technological advancements etc. Institutions plays an important role in the development of organization (Peng and Heath, 2007). Institutions are considered to be the 'rule of the game' present in a society which shape and constraint human behavior and interactions. They may differ across countries due to formal methods of collective decision making appointed e.g. authoritarian or democratic, or how power is distributed among various groups in an economy which produce different outcomes from a given arrangement of institution. The importance of different institutional setup is reflected in the divergence of growth rate between European nations and U.S. during 1990, when a rapid increase in technological growth rate resulted in the U.S. taking the lead due to its educational institutions which subsidized general education whereas, those in European nations aimed towards vocational education that reduced the flexibility in skills crucial in adapting new technology.

Similarly, institutions generate certain incentives to which people respond based on the cost and benefits associated with them (Boettke and Coyne, 2008). So that when the cost of certain behaviors rise e.g. rent seeking and tax evasion etc. people tend to move away from and when the cost associated reduces people engage more in those behaviors due to an increase in the benefits. Thus the analysis of growth of institutions start with the role of managers of the organization and how they are participating in the decision making process and their ability to make deliberate choices (Eisen and Schoonhoven, 1990).

This thesis will theoretically assess the role of organizational development with the help of growth model where there is accumulation of foreign technology in terms of Foreign Direct Investment (FDI) and physical capital in the growth of economy. Technology plays an important role in the process of economic growth. Technology spreading can take place through different instruments like adoption of foreign technology, accession of human capital, new ideas and foreign direct investments etc. Other than a source of technology transfer and financial development, foreign direct investment has other more important characteristics. It provides a variety of goods and services, improves the skills and knowledge of the managers and also it increases the productivity and efficiency of an economy.

Studies based on the neoclassical approach (Solow, 2005) argue that foreign direct investment affects only the level of income and the long-run growth remains unchanged. So, according to neo-classical models of economic growth, if technology grows permanently then it can be positively affected by FDI. Thus, according to neo-classical models of economic growth, FDI will only be growth advancing if it affects technology positively and permanently. Foreign direct investment has been known to play an important role in International Technology Transfer (IIT).The investments of multinationals to developing countries help to promote the technology transfer from developed nations to improve the available technology in their local firms (Glass and Saggi, 1997). Through this process i.e, International Technology Transfer the capacity of the organizations can be built up which will help in promoting the economic growth. Papers such as (Lin and Saggi, 1998), (Siotis, 1999), (Petit and Sanna-Randaccio, 2000), (Norback, 2001), (Glass and Saggi, 2002), (Bjorvatn and Eckel, 2006), (Sanna-Randaccio and Veugelers, 2007), and (Dawid et al., 2008) discuss FDI decisions in their relation to innovation and spillover.

Empirically, (Hoang et al., 2018) studied Vietnam to test the impact of FDI on economic growth, which resulted in significantly high growth in GDP from 4.4 per cent to 8.18 per cent in 1991-95. This increase in GDP also resulted an increase in average per capita of the individuals. This showed that the managers of Vietnam were planning better and managing the resources more efficiently. Likewise, in the empirical literature there are many papers that identify the positive relationship between foreign technology in terms of foreign direct investment and organizational development which eventually leads to the growth of the economy. Kugler (2000) investigate empirically that foreign direct investment (FDI) in a developing country generates positive externalities on local producers.

According to the literature survey data by Saggi (2002) the evidence supports the view that foreign direct investment has a positive externality on technology transfer in the host country hence accumulating economic growth.

In economics, the association between physical capital and development is contentious; difficult to measure issue (A. Szalavetz, 2005). Vinay (2016) has defined physical capital as:

"In economics, physical capital refers to a factor of production (or input into the process of production), such as machinery, buildings, or computers. In economic theory, physical capital is one of the three primary factors of production, also known as inputs production function."

Relating capital accumulation to productivity and income growth is a challenging task, not only because of difficulties in measuring capital as an input variable, but also because the importance of the relationship tends to change over time and space, as well as in line with the development. Greiner and Semmler (2001) analyze that there is a positive relationship between the investment in physical capital and development. Physical capital in terms of the accurate use of resources will help the organization to develop which will eventually lead to economic growth.

This thesis investigates the relationship between organizational development and economic growth through the accumulation of foreign technology and physical capital in a growth model. In theoretical literature, no one has considered organizational development in a growth model. Bucci and Marsiglio (2018) considered financial development in their growth model by formulating a Uzawa (1965)-Lucas (1988) type framework in combination with human and physical capital accumulation. In their model, financial development affect steady state growth by altering human and physical capital accumulation and such work has not been previously done in literature. I formulated my endogenous growth model similar to Bucci (Bucci and Marsiglio, 2018) where organizational development affect steady state growth through two sectors; foreign technology and physical capital.

The model will be solved in the standard mathematical procedures for the Balanced Growth Path (BGP) long-run equilibrium and steady state values, as is done in literature. Furthermore, the transitional dynamics and closed-for solution of the model will also be discussed. This thesis proceeds as follows: Chapter 1 will discuss the framework and the steady state solution of the model. Chapter 2 will discuss the closed form solution for the case of unique equilibrium where the share of physical capital is greater than the externality parameter. The functional forms for the growth rate of foreign technology and the adjustment cost is also discussed. Also the numerical simulations for the effect of change in the organizational development and the welfare effect is also observed. In Chapter 3 we have solved the model for the multiple equilibrium case of closed form solution where the share of physical capital is less than the externality parameter of foreign technology.

Chapter 1

Organizational development, physical capital, foreign technology and economic growth

This chapter builds up a two-sector organizational development endogenous growth model with physical capital and foreign technology. Moreover, the first order conditions are analyzed from Pontryagin's maximum principle. The externality in the production function gives more attention to the model. In addition, the steady state solution and non-negativeness conditions are established in this chapter.

1.1 Framework of the model

We have considered an economy where economic growth is the result of 'organizational development' in the form of accumulation of foreign technology and physical capital. The literature, it is well identified that economic growth can be affected by organizational development. For that reason, we have introduced foreign technology with an externality and physical capital in the production of the final good to make it more precise and generalizable. Bucci and Marsiglio (2018) has been used as a baseline model. For the utility maximization we will use current value Hamiltonian which is maximized subject to the foreign technology constraint along with the physical capital.

1.1.1 Production

Let $K(t)$ be the level of physical capital, $T(t)$ be the foreign technology, $T_A(t)$ be the average foreign technology and $u(t)$ be the efficiency of utilization of foreign technology with $0 < u < 1$. The model will be identified by the following Cobb-Douglas production function

$$Y = AK^{\alpha}(uT)^{1-\alpha}T_A^{\gamma} \tag{1.1}$$

where, α is the elasticity of output with respect to physical capital, A represents the constant technological level in final-goods sector, T_A^{γ} captures the external effect of foreign technology and parameter γ is a positive constant capturing the weight of the external effects. The production Y positively depends on physical capital K, the efficiency of utilization of foreign technology uT and on the stock of foreign technology $T_A(t)$. The term $T_A(t)$ appears as an externality in the production process. It represents the positive impact of foreign technology on the production of final domestic goods.

1.1.2 Utility function

We consider a closed and competitive economy which maximizes the utility function given as follows:

$$U(c(t)) = \frac{c^{1-\sigma}-1}{1-\sigma},\ \sigma \neq 1 \tag{1.2}$$

where $c(t)$ is the per capita consumption and σ is the inverse of intertemporal elasticity of substitution (IES).

1.1.3 Physical capital

The standard law of motion for the stock of physical capital is

$$\dot{K} = I(t) - \delta K(t), \tag{1.3}$$

where $I(t)$ is investment and δ is the depreciation rate of physical capital. For the sake of simplicity it is assumed that there is no physical capital depreciation. In this model, we assume that there is no population growth[1]. The resource constraint is:

$$Y(t) = c(t) + I(t), \tag{1.4}$$

Organizational capacity absorbs a share of $G(\theta)$ of income. The share of income lost in building physical capital lies between zero and one depending on the degree of organizational development.
After considering organizational development in the resource constraint, the economy's investment function is thus,

$$I = [1 - G(\theta)]Y - c, \tag{1.5}$$

By plugging the value of Y from (1.1), then (1.5) can be rewritten as,

$$I = [1 - G(\theta)]AK^{\alpha}(uT)^{1-\alpha}T_A^{\gamma} - c, \tag{1.6}$$

By combining equations (1.3) and (1.6), the final equation of physical capital accumulation is given by,

$$\dot{K} = A[1 - G(\theta)]K^{\alpha}(uT)^{1-\alpha}T_A^{\gamma} - c. \tag{1.7}$$

Physical capital accumulation is given by equation (1.7) where $G(\theta)$ is the share of income absorbed by the building up the organizational development and $0 < G(\theta) < 1$. More specifically, organizational development absorbs

[1]The population is normalized to one, so that all variables are expressed in per capita terms.

a share of income equal to $G(\theta)$. How organizational development affects this is not visible, but it is clear to believe that the more the organization is developed (the larger is θ), the less resources are wasted in the process of capacity building, that is $G'(\theta) < 0$.

Eisinger (2002) focused on hoe the key attributes of organizational development influence organizational effectiveness and mission attainment. Englander and Gurney (1994) discussed how the accumulation of human and physical capital which include infrastructure, technical knowledge, trade and R&D are considered as the main sources of growth in productivity in the long-term. Greiner and Semmler (2001) analyzed that there is a positive relationship between the investment in physical capital and development. Physical capital in terms of the accurate use of resources will help the organization to develop which will eventually lead to economic growth.

1.1.4 Foreign Technology

The foreign technology grows at a variable rate $B(\theta)$. Let a be the growth rate of foreign technology in the home country, $\overline{T}$ be the bounded foreign technology in the form of global technology frontier. The term $1-u$ represents the actual impact of foreign technology in capacity building. The equation of motion of foreign technology is given as

$$\dot{T}(t) = a\overline{T}B(\theta)(1-u)T. \tag{1.8}$$

$B(\theta)$ is the growth rate of foreign technology which depends on the degree of organizational development θ. Underlying this relationship is the intuition that the rate of adoption of foreign technology is dependent on the degree of organizational development, i.e., firms with better organizational structures find it easier to adopt foreign technology.

From the physical capital and foreign technology constraint it is clear that the degree of organizational development affects economic growth through two factors i.e, foreign technology and physical capital. An increase in the amount of Foreign Direct Investment will increase the production of final good which will ultimately develop the organization and hence economic growth.

1.2 The model

We have considered the Lucas-Uzawa (1988) two sector endogenous growth model with foreign technology and physical capital. A production externality is taken in the final good sector associated with the physical capital accumulation, under its standardized formulation as it was studied in Benhabib and Perli (1994) and Xie (1994). The economy is closed and competitive with the controls $c(t)$ and $u(t)$. We assume that agents maximize the utility function of the form

$$\text{Max}_{c,u} \quad \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t}, \ \sigma \neq 1 \tag{1.9}$$

subject to the constraints on the evolution of physical capital and foreign technology,

$$\dot{K} = A[1-G(\theta)]K^\alpha (uT)^{1-\alpha} T_A^\gamma - c, K_0 = K(0), \tag{1.10}$$

$$\dot{T} = a\overline{T}B(\theta)(1-u)T, T_0 = T(0). \tag{1.11}$$

Note that from equation (1.9) the optimizing representative agent takes T_A^γ as an exogenous function; but because of externality, the competitive equilibrium solution is discussed where $T_A = T$ which is different from the planner's solution.

1.3 First-order conditions

The current value Hamiltonian for this model is

$$
\begin{aligned}
H = \frac{c^{1-\sigma}-1}{1-\sigma} + \lambda\bigg[A[1-G(\theta)]K^{\alpha}(uT)^{1-\alpha}T_A^{\gamma} - c\bigg] \\
+\mu\bigg[a\overline{T}B(\theta)(1-u)T\bigg]
\end{aligned} \tag{1.12}
$$

The first-order necessary conditions are

$$\lambda = c^{-\sigma}, \tag{1.13}$$

$$\lambda A[1-G(\theta)]K^{\alpha}(1-\alpha)u^{-\alpha}T^{1-\alpha}T_A^{\gamma} = \mu a\overline{T}B(\theta)T, \tag{1.14}$$

$$\dot{\lambda} = \lambda\bigg[-A[1-G(\theta)]\alpha K^{\alpha-1}(uT)^{1-\alpha}T_A^{\gamma} + \rho\bigg], \tag{1.15}$$

$$\dot{\mu} = -\lambda A[1-G(\theta)]K^{\alpha}(1-\alpha)T^{-\alpha}u^{1-\alpha}T_A^{\gamma} - \mu a\overline{T}B(\theta)(1-u) + \rho\mu, \tag{1.16}$$

$$\dot{K} = A[1-G(\theta)]K^{\alpha}(uT)^{1-\alpha}T_A^{\gamma} - c, \tag{1.17}$$

$$\dot{T} = a\overline{T}B(\theta)(1-u)T, \tag{1.18}$$

The term T_A is taken as given in order to determine the competitive equilibrium (Naz and Chaudhry, 2016). Then, the first order conditions under the competitive conditions $T_A = T$ where all foreign technology are being treated identically, are

$$\lambda = c^{-\sigma}, \tag{1.19}$$

$$\lambda A[1-G(\theta)]K^{\alpha}(1-\alpha)u^{-\alpha}T^{1-\alpha+\gamma} = \mu a\overline{T}B(\theta)T, \tag{1.20}$$

$$\dot{\lambda} = \lambda\bigg[-A[1-G(\theta)]\alpha K^{\alpha-1}(u)^{1-\alpha}(T)^{1-\alpha+\gamma} + \rho\bigg], \tag{1.21}$$

$$\dot{\mu} = \mu[\rho - a\overline{T}B(\theta)], \tag{1.22}$$

$$\dot{K} = A[1-G(\theta)]K^{\alpha}u^{1-\alpha}T^{1-\alpha+\gamma} - c, \tag{1.23}$$

$$\dot{T} = a\overline{T}B(\theta)(1-u)T, \tag{1.24}$$

and the transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\lambda K(t) = 0, \tag{1.25}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu T(t) = 0. \tag{1.26}$$

Equations (1.19)- (1.20) yields following values of control variables

$$c = \lambda^{-\frac{1}{\sigma}} \tag{1.27}$$

$$u = \left(\frac{\mu a\overline{T}B(\theta)T^{\alpha-\gamma}}{\lambda A[1-G(\theta)]K^{\alpha}(1-\alpha)}\right)^{-\frac{1}{\alpha}}. \tag{1.28}$$

The time derivatives of (1.27) and (1.28) yield following growth rates of control variables (c, u):

$$\frac{\dot{c}}{c} = -\frac{1}{\sigma}[-A[1-G(\theta)]\alpha K^{\alpha-1}u^{1-\alpha}T^{1-\alpha+\gamma} + \rho] \tag{1.29}$$

$$\frac{\dot{u}}{u} = -\frac{c}{K} + \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u + \frac{a\overline{T}B(\theta)(1+\gamma-\alpha)}{\alpha}. \tag{1.30}$$

1.3.1 Sufficiency conditions

The constraint for T is non-concave due to the term $a\overline{T}B(\theta)uT$, therefore Mangasarian theorem fails. Sufficiency conditions are established through Arrow (1968) theorem in following proposition.

Proposition 1.1:

The first-order conditions are sufficient as well.

Proof:

In order to check for sufficiency conditions, the values of control variables[2] c and u which are given by $c = \lambda^{\frac{-1}{\sigma}}$ and $u = \left[\left(\frac{\lambda A[1-G(\theta)](1-\alpha)T_A^{\gamma}}{\mu a\overline{T}B(\theta)}\right)^{\frac{1}{\alpha}} T^{-1}K\right]$ can be substituted in the current value Hamiltonian to establish the maximized Hamiltonian. The maximized Hamiltonian is defined as

$$\begin{aligned} H^0(c,U,K,T,\lambda,\mu) = {} & \frac{\lambda^{\frac{\sigma-1}{\sigma}}-1}{1-\sigma} + K\left[\lambda A[1-G(\theta)]T_A^{\gamma}\right]^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\mu a\overline{T}B(\theta)}\right)^{\frac{1-\alpha}{\alpha}} - \lambda^{\frac{\sigma-1}{\sigma}} \\ & + T\mu a\overline{T}B(\theta) - K\left[\mu a\overline{T}B(\theta)\right]^{\frac{\alpha-1}{\alpha}}\left[\lambda A[1-G(\theta)](1-\alpha)T_A^{\gamma}\right]^{\frac{1}{\alpha}} \end{aligned} \tag{1.31}$$

[2]Here, we are taking the values of control variables c and u where $T \neq T_A$.

The maximized Hamiltonian is linear in state variables K and T and thus it is always concave in state variables. Therefore, we conclude that the first-order conditions are sufficient by Arrow's theorem. This completes proof of proposition 1.1.

1.4 Steady state solution

A balanced growth path is a sequence of time path along which all economic variables (c, u, K, T) grow at a constant rate. For that, we will solve the model for steady state values. The solution does not exist for the original values so we will use a dimensionality reduction technique by taking ratios of variables. We will use this technique followed by Mulligan and Sala-I-Martin(1993). Therefore, we can study the dynamics of simplified system by introducing the variables, $\chi = \frac{c}{K}$, $\psi = \frac{K}{T^{\frac{1-\alpha+\gamma}{1-\alpha}}}$. The time derivatives of these variables are as

$$\frac{\dot{\chi}}{\chi} = \frac{\dot{c}}{c} - \frac{\dot{K}}{K}, \tag{1.32}$$

$$\frac{\dot{\psi}}{\psi} = \frac{\dot{K}}{K} - \left(\frac{1-\alpha+\gamma}{1-\alpha}\right)\frac{\dot{T}}{T}. \tag{1.33}$$

With the aid of equations (1.23) and (1.24), equations (1.32) and (1.33) can be re written as follows

$$\frac{\dot{\chi}}{\chi} = A[1-G(\theta)]\left(\frac{\alpha}{\sigma}-1\right)u^{1-\alpha}\left(\frac{K}{T^{\frac{1-\alpha+\gamma}{1-\alpha}}}\right)^{\alpha-1} + \frac{c}{K} - \frac{\rho}{\sigma}, \tag{1.34}$$

$$\frac{\dot{\psi}}{\psi} = A[1-G(\theta)]K^{\alpha-1}u^{1-\alpha}T^{1-\alpha+\gamma} - \frac{c}{K} - \left(\frac{1-\alpha+\gamma}{1-\alpha}\right)a\overline{T}B(\theta)(1-u). \tag{1.35}$$

By using the variables $\chi = \frac{c}{K}, \psi = \frac{K}{T^{\frac{1-\alpha+\gamma}{1-\alpha}}}$ in equation (1.34) and (1.35) we can derive the following three dimensional system

$$\frac{\dot{\chi}}{\chi} = A[1-G(\theta)]\left(\frac{\alpha}{\sigma}-1\right)u^{1-\alpha}\psi^{\alpha-1} + \chi - \frac{\rho}{\sigma}, \tag{1.36}$$

$$\frac{\dot{\psi}}{\psi} = A[1-G(\theta)]u^{1-\alpha}\psi^{\alpha-1} - \chi - \frac{1-\alpha+\gamma}{1-\alpha}a\overline{T}B(\theta)(1-u), \tag{1.37}$$

$$\frac{\dot{u}}{u} = -\chi + \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u + \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{\alpha}. \tag{1.38}$$

The existence and non-negativeness of the steady state values is established in the following propositions.

Proposition 1.2:

The steady state solution exist for all set of variables (χ^*, ψ^*, u^*) and are given as,

$$\chi^* = \frac{a\overline{T}B(\theta)}{\alpha} + \frac{(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho)}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]}, \tag{1.39}$$

$$u^* = 1 - \frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]}, \tag{1.40}$$

$$\psi^* = \left[\frac{\sigma(1-\alpha+\gamma)a\overline{T}B(\theta)-\gamma\rho}{A[1-G(\theta)]\alpha[(1-\alpha+\gamma)\sigma-\gamma]}\right]^{\frac{1}{\alpha-1}} u. \tag{1.41}$$

Proof:

The complete proof for proposition 1.2 is presented in Appendix A1.

Proposition 1.3:

The parameter space for which $0 < u < 1$, is specified by the following two regions,
$\Omega_1 = \left[a\overline{T}B(\theta) > \rho, \gamma > 0, \sigma > \sigma_m\right]$, $\Omega_2 = \left[a\overline{T}B(\theta) < \rho < \rho_m, \gamma > 0, 0 < \sigma < \sigma_m\right]$ provided $a\overline{T}B(\theta) - \rho$ and $\sigma(1-\alpha+\gamma)-\gamma$ have same sign.
where

$$\sigma_m = 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$$

and,

$$\rho_m = \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{1-\alpha}.$$

Proof:

The complete proof is presented in Appendix A2.

Proposition 1.4:

i. For $\gamma > \alpha$, χ is positive as $a\overline{T}B(\theta) - \rho$ and $\sigma(1 - \alpha + \gamma) - \gamma$ have same sign from proposition 1.3.

ii. For $\gamma < \alpha$, the variable $\chi > 0$ in region Ω_1 provided $\sigma_m > \sigma_n$ and in region Ω_2 provided $\sigma_m < \sigma_n$.

where

$$\sigma_m = 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$$

and

$$\sigma_n = \alpha - \frac{\rho(1-\alpha)(\alpha-\gamma)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$$

Proof:

See Appendix A3 for complete proof.

Proposition 1.5:

The balance growth path is characterized by a strict positive level of consumption, physical capital and foreign technology to the production of final good. The growth rate of consumption g_c, growth rate of foreign technology g_T and the growth rate of physical capital g_K are given as follows

$$g_c = g_K = \frac{(1 - \alpha + \gamma)(a\overline{T}B(\theta) - \rho)}{\sigma(1 - \alpha + \gamma) - \gamma} \tag{1.42}$$

$$g_T = \frac{(1 - \alpha)(a\overline{T}B(\theta) - \rho)}{\sigma(1 - \alpha + \gamma) - \gamma} \tag{1.43}$$

whereas, the growth rate of efficiency of utilization of foreign technology $g_u = 0$. Moreover, the growth rates of consumption g_c, physical capital g_K and foreign technology g_T are positive provided $a\overline{T}B(\theta) - \rho$ and $\sigma(1 - \alpha + \gamma) - \gamma$ have same sign.

Proof:

See Appendix A4 for detailed proof of proposition 1.5.

1.5 Conclusion

This chapter presented a two sector endogenous growth model with foreign technology and physical capital. It builds a theoretical framework by incorporating organizational development in both two sectors. Firstly, the first order conditions are analyzed for the competitive equilibrium condition where $T_A = T$. Secondly, the sufficiency conditions are checked through Arrow's sufficiency theorem. Then, through the dimensionality reduction technique we introduced two new variables by taking the ratios of the original variables. In addition, from the existence and non-negativeness conditions it has proven that unique steady state solution exists for our model.

Chapter 2

The closed form solution for the case of unique equilibrium

After showing the steady state solutions and stability analysis, we now measure the welfare effects of organizational development through the analysis of the transitional dynamics of our model. Since (1.36), (1.37), and (1.38) form a simultaneous system of differential equations, analyzing its transitional behavior is evidently not possible in analytical terms. By considering a special case it is feasible to split some of these equations and then solve the system, which will eventually allow us to examine the welfare effects of an organizational development. This is only plausible by solving the model through the closed form solution when $\sigma = \alpha$, that is whenever the inverse of the intertemporal elasticity of substitution is equal to the physical capital share (Xie, 1994). In this case it is possible to show that the following result holds:

2.1 The Case $\sigma = \alpha > \gamma$

Proposition 2.1: A unique non-explosive path exists for χ. When $\sigma = \alpha$ along which χ remains constant at the equilibrium value $\chi = \frac{\rho}{\alpha}$. Moreover, this yields a particular and well defined value for control variable $c_0 = \frac{\rho}{\sigma} K_0$ where $\sigma = \alpha$.

Proof:

From (1.36), under the assumption $\sigma = \alpha$, we have,

$$\dot{\chi} + \frac{\rho \chi}{\alpha} = \chi^2 \tag{2.1}$$

which is Bernoulli's differential equation. By solving (2.1) subject to initial condition $\chi(0) = \chi_0$ through Bernoulli's technique, we get

$$\chi(t) = \frac{(\frac{\rho}{\alpha})\chi_0}{\chi_0 + \left(\frac{\rho}{\alpha} - \chi_0\right) e^{\frac{\rho}{\alpha} t}}. \tag{2.2}$$

The trasversality condition (1.25) for K can be written as

$$\lim_{t\to\infty} \lambda K e^{-\rho t} = \lim_{t\to\infty} \frac{\lambda^{\frac{\alpha-1}{\alpha}}}{(\frac{\rho}{\alpha})} e^{-\rho t} + \lim_{t\to\infty} \lambda^{\frac{\alpha-1}{\alpha}} \frac{((\frac{\rho}{\alpha}) - \chi_0)}{(\frac{\rho}{\alpha})\chi_0} e^{(\frac{\rho}{\alpha} - \rho)t} \tag{2.3}$$

The transversality imposes a necessary but not a sufficient condition that $\lim_{t\to\infty} \lambda^{\frac{\alpha-1}{\alpha}} e^{-\rho t} = 0$ for first term and $\chi_0 = \frac{\rho}{\alpha}$ for the second term. Thus equation (2.2) yields

$$\chi(t) = \frac{\rho}{\alpha}. \tag{2.4}$$

This results in a particular and well defined value for control variable c.

$$c_0 = \frac{\rho}{\sigma} K_0. \tag{2.5}$$

This completes the proof. The step by step calculations of proof of Proposition 2.1 are presented in detail in Appendix B1.

Proposition 2.2: Under the competitive equilibrium conditions, a unique non-explosive path for u along which u remains constant at the equilibrium value $u = 1 - \frac{a\overline{T}B(\theta)-\rho}{a\overline{T}B(\theta)(\alpha-\gamma)} = u^*$ if and only if $\gamma < \alpha$. The value of u lies between $0 < u < 1$ if and only if $a\overline{T}B(\theta)(1-\alpha+\gamma) < \rho < a\overline{T}B(\theta)$.

Proof:

By plugging the value of χ from equation (2.4) in equation (1.38), we have

$$\dot{u} + \frac{\rho - a\overline{T}B(\theta)(1-\alpha+\gamma)}{\alpha}u = \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u^2, \tag{2.6}$$

The solution of equation (2.1) subject to initial condition $u(0) = u_0$ through Bernoulli's technique is given by

$$u(t) = \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0}, \tag{2.7}$$

where $u^* = \frac{\rho - a\overline{T}B(\theta)(1-\alpha+\gamma)}{a\overline{T}B(\theta)(\alpha-\gamma)}$. Now

$$\lim_{t\to\infty} u(t) = \lim_{t\to\infty} \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0} \tag{2.8}$$

which attains a unique equilibrium value u^* provided $a\overline{T}B(\theta)(\alpha-\gamma)[u^*-u_0] = 0$ and $\gamma < \alpha$. Thus equation (2.7) results in

$$u(t) = 1 - \frac{a\overline{T}B(\theta) - \rho}{a\overline{T}B(\theta)(\alpha-\gamma)} = u^*. \tag{2.9}$$

Now $u > 0$ provides $a\overline{T}B(\theta)(1-\alpha+\gamma) < \rho$ and $u < 1$ yields $\rho < a\overline{T}B(\theta)$. This completes the proof of proposition.

The step by step calculations of proof of Proposition 2.2 are presented in detail in Appendix B2.

The restriction $\gamma > \alpha$ generates indeterminacy and will be discussed in chapter 3.

Proposition 2.3: Under the parameter restrictions $\gamma < \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) < \rho < a\overline{T}B(\theta)$, the foreign technology $T(t)$ satisfying the trasversality condition (1.26) is given by

$$T(t) = T_0 e^{(\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma})t}. \tag{2.10}$$

Proof:

Equation (1.24) results in

$$\dot{T} = \left(\frac{a\overline{T}B(\theta) - \rho}{\alpha - \gamma}\right)T. \tag{2.11}$$

Equation (2.11) is a linear ordinary differential equation for T and yields solution given in (2.10). The solution for co-state variable μ for equation (1.22) is $\mu = \mu_0 e^{(\rho - a\overline{T}B(\theta))t}$. Consequently, the trasversality condition (1.26) for $T(t)$ gives:

$$\lim_{t\to\infty} T\mu e^{-\rho t} = \mu_0 e^{(\rho - a\overline{T}B(\theta))t} e^{-\rho t} T_0 e^{(\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma})t} \tag{2.12}$$

goes to zero provided $a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho < 0$. This completes the proof of proposition.

The step by step calculations of proof of Proposition 2.3 are presented in detail in Appendix B3.

Proposition 2.4: Under the parameter restrictions $\gamma < \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) < \rho < a\overline{T}B(\theta)$, the physical capital $K(t)$ satisfying the trasversality condition (1.25) is given by

$$K(t) = e^{-\frac{\rho}{\alpha}t}\left[K_0^{1-\alpha} + \phi\left(e^{\frac{(a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma)t}{\alpha(\alpha-\gamma)}} - 1\right)\right]^{\frac{1}{1-\alpha}} \tag{2.13}$$

where,
$\phi = \frac{\alpha(\alpha-\gamma)(1-\alpha)A[1-G(\theta)]u^{*1-\alpha}T_0^{1-\alpha+\gamma}}{a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma}$
Moreover,

$$\tfrac{c}{K} = \tfrac{\rho}{\alpha} \Longrightarrow c(t) = \tfrac{\rho}{\alpha}K(t).$$

Proof:

Using $c = \frac{\rho}{\alpha}K$ in equation (1.23) results in

$$\dot{K} + \frac{\rho}{\alpha}K = A[1-G(\theta)]u^{*1-\alpha}T_0^{1-\alpha+\gamma}e^{\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma}(1-\alpha+\gamma)t}K^{\alpha} \tag{2.14}$$

By solving equation (2.14) through Bernoulli's differential technique, yields the solution given in (2.13). As a result, the trasversality condition (1.25) for $K(t)$ gives:

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty} \left[\frac{\rho}{\alpha}\right]^{-\alpha} \left[K_0^{1-\alpha} + \phi\left(e^{\frac{a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma}{\alpha(\alpha-\gamma)}t} - 1\right)\right] e^{-\frac{\rho}{\alpha}(1-\alpha)t} e^{-\rho t} \tag{2.15}$$

goes to zero provided, $a\overline{T}B(\theta)(1-\alpha+\gamma) > \frac{\rho}{\alpha}\gamma$..

The step by step calculations of proof of Proposition 2.4 are presented in detail in Appendix B4.

Proposition 2.5:

i) The dynamic growth rate of foreign technology T attains the following equilibrium value in long run

Equation (2.10) yields

$$T(t) = T_0 e^{(\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma})t}$$

Taking limit on both sides, we have

$$\lim_{t\to\infty} T\mu e^{-\rho t} = \mu_0 e^{(\rho - a\overline{T}B(\theta))t} e^{-\rho t} T_0 e^{(\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma})t} \tag{2.16}$$

$$g_T = \frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma} > 0$$

provided $(a\overline{T}B(\theta) - \rho) > 0$.

ii) The dynamic growth rate of physical capital K by utilizing equation (2.14) is,

$$g_K = \frac{(a\overline{T}B(\theta)-\rho)(1-\alpha+\gamma)}{(1-\alpha)(\alpha-\gamma)} > 0$$

provided $a\overline{T}B(\theta)(1-\alpha+\gamma) > \frac{\rho}{\alpha}\gamma$ in long run. Moreover, the unique equilibrium value of c starting from $c_0 = \frac{\rho}{\alpha}K_0$, shows the transitional dynamics and approaches asymptotically the unique balanced growth path with the rate $g_c = g_K$.

2.2 The Functional Forms

According to Harbison F. (1956), the concept of organizational development is most useful in determining the economic growth of an economy. In our model, organizational development affects the BGP equilibrium through two functions: the growth rate of foreign technology $B(\theta)$ and the share of income $G(\theta)$.

2.2.1 The functional form of growth rate of foreign technology

The objective of this thesis is to explain the economic growth by building a two sector endogenous growth model by taking into account physical capital and foreign technology. The foreign technology grows at the rate of $B(\theta)$ which is the function of an organizational development. The positive relationship between growth rate and organizational development holds true if $B^{'}(\theta) > 0$. Therefore, we'll take simple linear form in which $B(\theta)$ is a linear function of θ:

$$B(\theta) = a_1\theta \tag{2.17}$$

here, $B^{'}(\theta)$ is also greater than zero (provided $a_1 > 0$) and reinforces strictly positive and monotonic relationship of organizational development and economic growth (illustrated in Figure (2.1)).
The figure clearly shows that the positive and monotonic relationship exists between growth rate of foreign technology and development of an organization. This is also supported by the empirical literature where it is observed that the growth of foreign technology in terms of Foreign Direct Investment (FDI) is expected to boost host economic growth and maintain macroeconomic stability (Zhang, K. H. 2001). "Although some of the evidence from the literature shows that the growth consequences of foreign technology differ by country of origin, and that these country of origin effects also vary depending on the

host country characteristics (Fortanier, F. 2007). Zhang and Zou (1995) investigate the relationship between foreign technology imports and economic growth in developing countries. They develop an intertemporal endogenous growth model that explicitly accepts foreign technology imports as a factor of production. The model establishes a link between the growth rate of productivity in a developing country and the country's intensity of learning to use foreign technologies. They hypothesize that a developing country's economic growth rate increases as foreign technology imports increase. They run regressions with data for about 50 developing countries, using different econometric methods and time spans. These empirical tests confirm the hypothesis that foreign technology transfers boost income growth rates. Moreover, economic developing in developing countries differs from that in industrial countries. In developing countries, increases in productivity depend not on innovation but on importing foreign plants and equipment and on borrowing foreign technology. Therefore, the well developed organization have positive relationship with growth rate of foreign technology of an economy and hence contribute to economic growth".

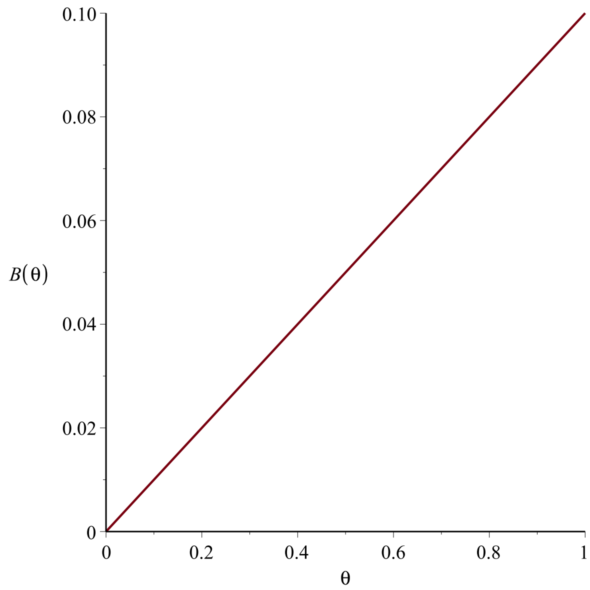

Figure 2.1: Graph for $B(\theta)$

2.2.2 The functional form of adjustment cost

Organizational development negatively impacts the adjustment cost by a variable $G(\theta)$. Drazen and Eckstein (1988) show that in a simple dual economy model capital accumulation and aggregate income will be lowest when both factor markets in agriculture are fully competitive. This empirical evidence supports our findings that if organization is fully developed, it will consume a smaller share of income exhibiting negative relationship. Therefore, we'll assume exponential form in which $G(\theta)$ is an exponential function of θ:

$$G(\theta) = a_2 e^{-\theta} \tag{2.18}$$

here, $G^{'}(\theta)$ is also less than zero and reinforces strictly negative relationship of organizational development and the adjustment cost (illustrated in Figure (2.2)).

The graph of $G(\theta)$ is negatively sloped which is clearly showing the negative relationship between the adjustment cost and organizational development. In the study of India (Bloom et al., 2013) they ran a field experiment in which they provide free management training to randomly chosen treatment plants and compare them with the control plants. They saw that these practices improved management techniques which resulted an improvement in productivity of 17 percent. The literature also supports this finding that if organization is fully it will absorb the less share of total income and vice versa.

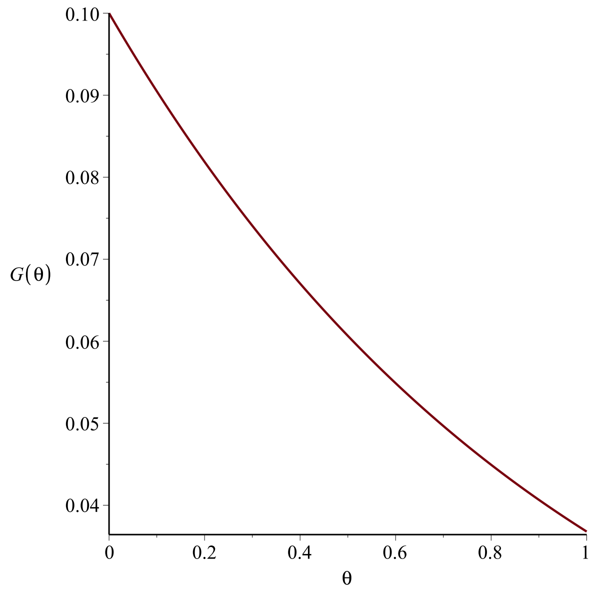

Figure 2.2: Graph for $G(\theta)$

2.3 Evolution over time of consumption, physical capital, foreign technology and efficiency of utilization of foreign technology

This section will discuss the time evolution on BGP equilibrium of consumption, utilization of foreign technology, physical capital and foreign technology. Also the functional forms for the growth rate of foreign technology and the share of income will be discussed further. Organizational development plays a central role in determining economic growth and BGP equilibrium. In order to analyze the BGP equilibrium we consider the benchmark values for the key parameters as follows in Table (2.1) [La Torre and Marsiglio (2010); Chaudhry et al. (2017); Bucci and Marsiglio (2018)].

Table 2.1: Parameter values employed in our simulation

$\sigma = \alpha$	ρ	γ	K_0	T_0	u_0	$\overline{T}$	A	a	a_1	a_2	θ
0.35	0.03	0.13	10	10	0.71	2	1	0.2	0.1	0.1	0.8

Figure (2.3) presents the result of our simulation exercise for different values of time(t). As it is clear from the graph that the trend for consumption $c(t)$, physical capital $K(t)$ and foreign technology $T(t)$ over time are similar i.e, positively sloped. The graph for consumption $c(t)$ shows that there is an increase in the consumption of the final good as we move forward over time. Initially, in the short-run there is less consumption but as time passes the consumption is increasing. The graph for the efficiency of utilization of foreign technology $u(t)$ is horizontal, showing that with respect to time (t), the utilization is not changing i.e, it remains constant.

From figure it is obvious that the physical capital $K(t)$ and foreign technology $T(t)$ are positively related with time (t). At time period 20 i.e, in the short-run there is a small increase in physical capital $K(t)$ and foreign technology $T(t)$. As we move forward there is an instant growth in long-run.

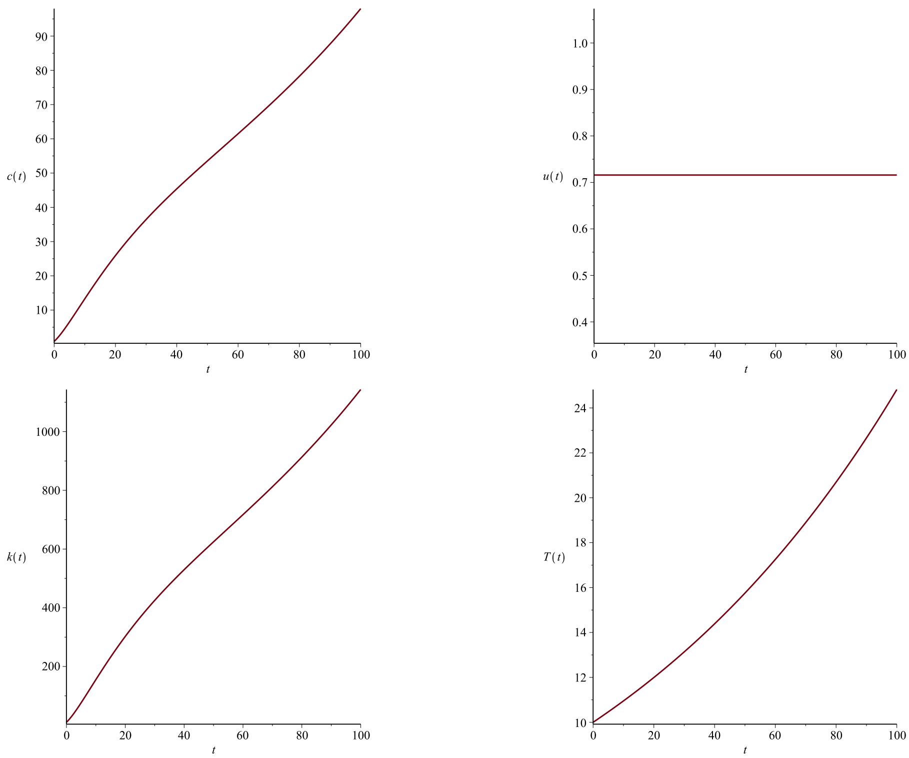

Figure 2.3: Evolution over time of consumption, efficiency rate of foreign technology, physical capital and foreign technology.

2.4 Effects of Change in Organizational Development $'\theta'$ on our key variables of the model

The effect of change in the development of organizational capacity (θ) on BGP equilibrium with respect to change in time (t) is shown in Figure (2.4). In the figure all key variables are positively related to the organizational development except the efficiency of utilization of foreign technology i.e, $u(t)$. The trend for per capita consumption $c(t)$ and physical capital $K(t)$ are almost similar. An increase in the level of organizational development(θ) will decrease both consumption and physical capital in the short run but it will increase both in the long run period. At the benchmark value of θ at 0.80, both $c(t)$ and $K(t)$ are increasing at an increasing rate. The magnitude of θ is increasing after time period t at 50 as shown in the figure; which shows that an increase in the organizational capacity has a larger (positive) impact on the per capita consumption and physical capital.

If we go below the benchmark value the effect of $c(t)$ and $K(t)$ increases but a decreasing rate. For the figure $u(t)$ it is clear that the efficiency of utilization of foreign technology remains the same with respect to time t. This is also proven in our proposition 2.2. As θ increases the utilization decreases but at a constant rate. We can also see that above the benchmark value; the effect almost goes to zero. For the graph of $T(t)$; it is an increasing function of time t both in the short and long run period. The effect of change in θ above the benchmark value is larger as we move further with respect to time and it almost approaches to zero below the benchmark value of organizational development. Such an increase is due to the fact that a higher degree of an organizational development θ tends to decrease the share of foreign technology T allocation to the production activities, thus increasing the growth rate of foreign technology. The last graph is showing the effect of change in the organizational development on the production function. The trend is almost the same as $c(t)$ and $K(t)$. The production function is decreasing in the short run but increasing in the long run.

An improvement in an organizational development leads to the greater impact on foreign technology because foreigners perceive that they will get greater returns. Also an increase in organizational development will lead to increase in capital accumulation which will hence lead to greater growth. Thus, high foreign technology and greater capital; there will be an increase in the income. Eventually, high income leads to high consumption of goods and services.

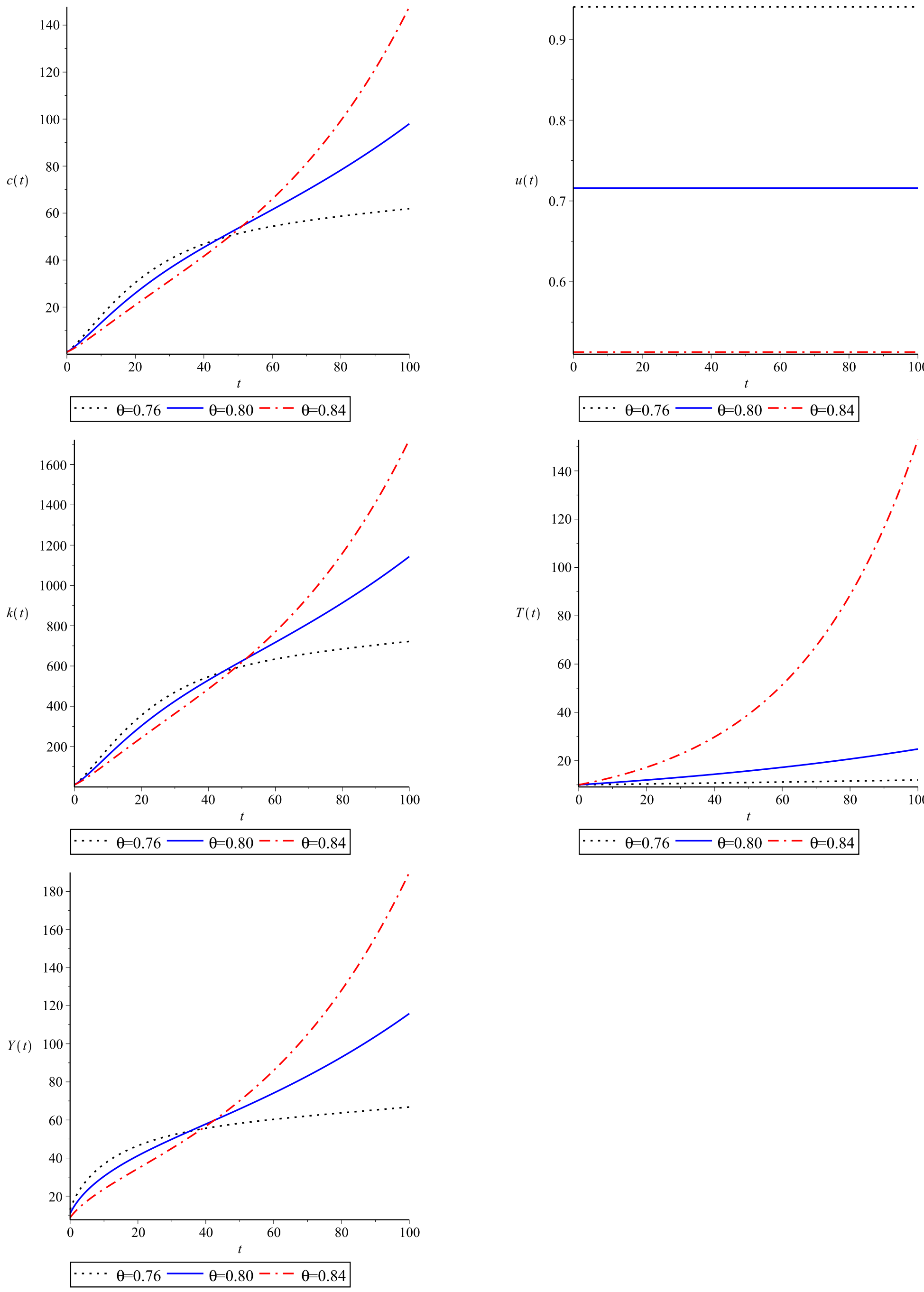

Figure 2.4: Evolution over time of $c(t), u(t), k(t), T(t)$ and Y for different values of organizational development

2.5 Conclusion

In this chapter we have analyzed the uniqueness of equilibrium path of special case of $\sigma = \alpha$, that is whenever the inverse of inter-temporal elasticity of substitution coincides with the physical capital share (Xie, 1994). Then we have discussed that organizational development impacts economic growth through two factors i.e, growth rate of foreign technology $B(\theta)$ and share of income $G(\theta)$. For this we have determined functional forms for these two variables and also justified with the help of empirical evidences. After that we have generated some numerical simulations over different values of our key variables. In this regard, we looked upon the evolution of time over consumption, share of physical capital, foreign technology and efficiency of utilization of foreign technology. Secondly, we looked at the effect of change in organizational development on all variables i.e, $c(t)$, $K(t)$, $T(t)$ $u(t)$ and Y.

Chapter 3

The closed form solution for the case of multiple trajectories

In this chapter we will discuss the second case for the closed-form solutions where the share of physical capital is less than the inter-temporal elasticity of substitution. In this setting there exists a continuum of equilibrium paths starting from the initial endowments of the foreign technology and physical capital. These equilibrium paths can be indicated by the value of u_0. A country with lower endowments of physical capital and foreign technology can have higher steady state value as long as it has smaller u_0. This means that the country with lower endowments can overtake the richer country at some point in time. That is, the country with an initial higher endowments will start below but will finish first. The following propositions are the list of the conclusions that may reflect the explicit solution.

3.1 The Case $\sigma = \alpha < \gamma$

Proposition 3.1: Under the competitive equilibrium conditions if $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho > 0$ there exists a continuum of equilibrium paths for c starting from $c_0 = \frac{\rho}{\alpha}K_0$.

Proof:

Given (1.19) and Proposition 2.1 we get,

$$c = \frac{\rho}{\alpha}K \tag{3.1}$$

Proposition 3.2: Under the competitive equilibrium conditions if $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho > 0$ there exists a continuum of equilibrium paths for u.

Proof:

Given Proposition 2.2, from equation (2.7), the value of control variable u can be reduced to

$$u(t) = \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0}, \tag{3.2}$$

where

$$u^* = \frac{\rho - a\overline{T}B(\theta)(1-\alpha+\gamma)}{a\overline{T}B(\theta)(\alpha-\gamma)} \tag{3.3}$$

, Taking limit on both sides

$$\lim_{t\to\infty} u(t) = \lim_{t\to\infty} \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0}. \tag{3.4}$$

$\lim_{t\to\infty} u(t) = u^*$, as $\alpha - \gamma < 0$ and $a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho > 0$. Equation (3.2) gives a spectrum of solution trajectories for u because of the indeterminacy.

Proposition 3.3: Under the competitive equilibrium conditions

(i) If $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho > 0$ there exists a spectrum of equilibrium paths for T starting from T_0 which is given by,

$$T = T_0\left[1 - \frac{u_0}{u^*} + \frac{u_0}{u^*}e^{-\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}\right]^{\frac{\alpha}{\alpha-\gamma}} e^{a\overline{T}B(\theta)t} \tag{3.5}$$

these paths are classified by the multiplicity of initial values of u starting from u_0 given by,

$$u(0) = \frac{u_0}{F(t)} e^{\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t} \tag{3.6}$$

where,

$$F(t) = \frac{u^* - u_0 + u_0 e^{-\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}}{u^*} \tag{3.7}$$

(ii) If $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho \leq 0$ then no equilibrium path exist for T starting from T_0.

Proof:

From equation (1.24)

$$\dot{T} = a\overline{T}B(\theta)(1-u^*)T \tag{3.8}$$

where,

$$u^* = \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0} \tag{3.9}$$

Equation (3.8) is a linear differential equation for T and provides a solution given in (3.5). Therefore, the transversality condition (1.26) for $T(t)$ gives

$$\lim_{t\to\infty} T(t)u(t)e^{-\rho t} = \lim_{t\to\infty} T_0 F(t)^{\frac{\alpha}{\alpha-\gamma}} e^{(a\overline{T}B(\theta)-\rho)t} \left[u_0 e^{(\rho - a\overline{T}B(\theta))t} \right] \tag{3.10}$$

goes to zero provided[1] $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho > 0$.

The complete proof is presented in Appendix C1.

Proposition 3.4: Under the competitive equilibrium conditions If $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho > 0$ there exists a sequence of equilibrium paths for K starting from K_0 satisfying the transversality condition which is given by,

$$K = \left[K_0^{1-\alpha} + A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t) \right]^{\frac{1}{1-\alpha}} e^{-\frac{\rho}{\alpha}t} \tag{3.11}$$

[1]Note that: $\lim_{t\to\infty} F(t) = \frac{u^*-u_0}{u^*} + \frac{u_0}{u^*}e^{\infty} = \infty$

where,

$$Z(t) = \int F^{\frac{\gamma}{\alpha-\gamma}} e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} dt \tag{3.12}$$

these paths are classified by the multiplicity of initial values of u starting from u_0.

Proof:

Using the value of u from (3.6) and T from (3.5) and $c = \frac{\rho}{\alpha}K$, equation (1.32) results in,

$$\dot{K} + \frac{\rho}{\alpha}K = A[1 - G(\theta)]u_0^{1-\alpha}T_0^{1-\alpha+\gamma}F^{\frac{\gamma}{\alpha-\gamma}} e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t-(1-\alpha)\frac{\rho}{\alpha}t}K_0^{\alpha} \tag{3.13}$$

By solving equation (3.13) by Bernoulli's differential technique yields the solution given in (3.11). Therefore, the transversality condition (1.25) for $K(t)$ gives,

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty}\left(\frac{c_0}{K_0}\right)^{-\alpha}\left[K_0^{1-\alpha} + A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t)\right]e^{-\frac{\rho}{\alpha}(1-\alpha)t}e^{-\rho t} \tag{3.14}$$

Note that, $\lim_{t\to\infty} Z(t) = \infty$

$$Z(t) = \int F^{\frac{\gamma}{\alpha-\gamma}} e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} dt \tag{3.15}$$

$$\lim_{t\to\infty} F(t)^{\frac{\gamma}{\alpha-\gamma}} e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} = \lim_{t\to\infty} \frac{u_0}{u^*} e^{\left(\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{\gamma} - \frac{\rho}{\alpha}\right)t} \tag{3.16}$$

goes to zero provided $a\overline{T}B(\theta)(1-\alpha+\gamma) < \frac{\gamma\rho}{\alpha}$.

The complete proof is presented in Appendix C2.

Proposition 3.5: A country with lower endowments of physical capital and foreign technology can attain higher level of production in the long run as compared to the country that has larger endowments, provided that former considers larger initial value of the efficiency of the utilization of foreign technology in the production of goods than the latter.

Proof:

We know that both countries will eventually converge at the same rate. From equations (3.1) and (3.9), the country with lower endowments of physical capital and foreign technology can have higher steady state value as long as it has larger u_0. This means that the country with lower endowments will overtake the richer country at some point in time. That is, the country with an initial higher endowments will start below but will finish first.

3.2 Effects of change in organizational development $'\theta'$ on foreign technology and the efficiency of utilization of foreign technology

The effect of change in the level of organizational development on foreign technology $T(t)$ and the efficiency of utilization of foreign technology $u(t)$ is shown in the figure below. In order to analyze the BGP equilibrium I have consider the benchmark values for the key parameters as follows in Table (3.1) [La Torre and Marsiglio (2010); Chaudhry et al. (2017); Bucci and Marsiglio (2018)].

Table 3.1: Parameter values employed in our simulation

$\sigma = \alpha$	ρ	γ	K_0	T_0	u_0	$\overline{T}$	A	a	a_1	a_2	θ
0.35	0.03	0.40	10	10	0.35	2	1	0.2	0.1	0.1	0.73

The foreign technology is positively related to the organizational development. An increase in the organizational development tends to increase the foreign technology in the home country which means that if an economy is developed; it will hep them to attract more foreign technology which will ultimately make them more developed. Similarly, if there is a decrease in organizational development the foreign technology also goes down. So the foreign technology is decreasing in the short run but in the long run it increases sharply. The second graph is showing the negative relationship between the efficiency of utilization of foreign technology and organizational development. But it is clear that an increase in the level of organizational development will shift the graph upwards and vice versa.

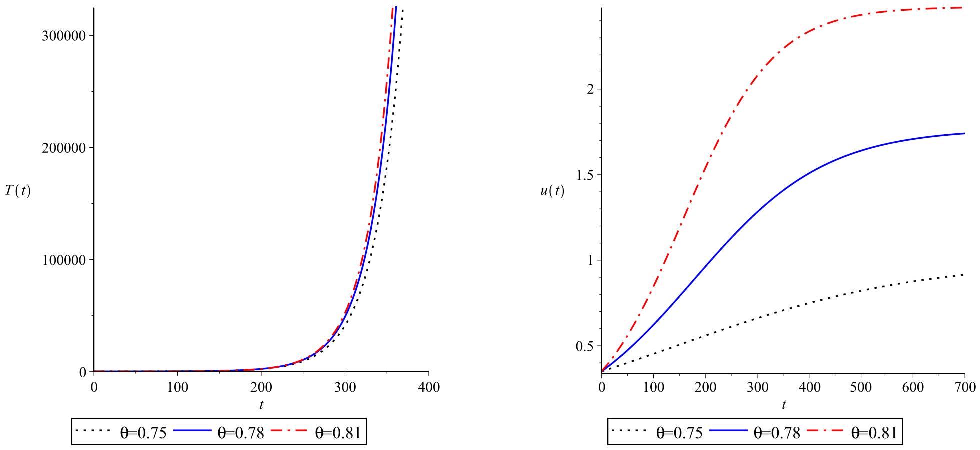

Figure 3.1: Evolution over time of $u(t)$ and $T(t)$ for different values of organizational development

3.3 Transitional dynamics of foreign technology and the efficiency of utilization of foreign technology

As we have discussed in the above sections that if the difference between the effect of externality of foreign technology and the share of physical capital is positive i.e, $\gamma - \alpha > 0$; there will be the multiple paths, each of them will converge to the different equilibrium values. It is clear from Figure 3.2[2], the equilibrium value of foreign technology $T(t)$ is a decreasing function with respect to the value of u_0. This can be explained by the cross country comparison. Let the country A be endowed with the greater efficiency of utilization of foreign technology as compared to the country B. Then as long as the country A has a higher production; they will accumulate less of their resources. The higher the of efficiency of utilization of foreign technology; less number of foreign technology will be used in the production of goods and services. Whereas, the variable $u(t)$ has the same constant equilibrium value. Hence the country

[2]The simulations are performed at the benchmark values given in Table 3.1

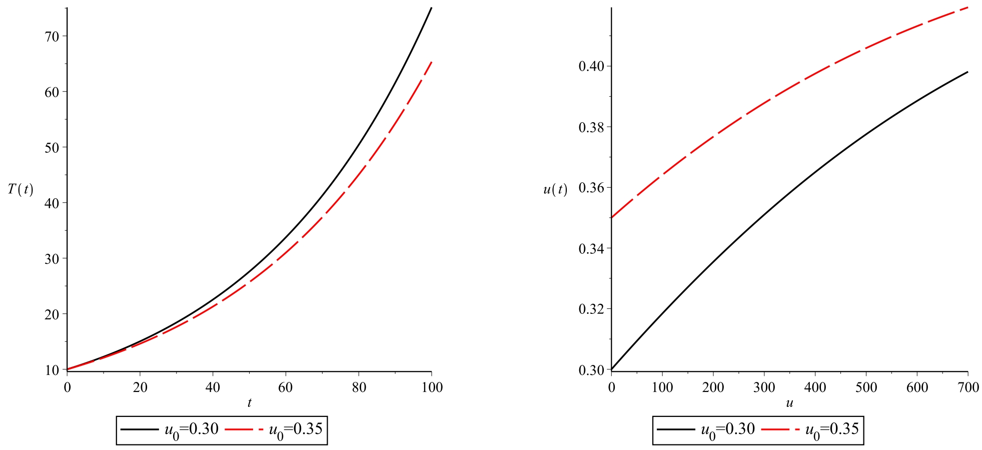

Figure 3.2: Transitional dynamics for $T(t)$ and $u(t)$

A will overtake the country B.

3.4 Conclusion

In this chapter we have discussed that what happens if the effect of externality is greater than the share of physical capital in the production function. For the existence of multiple equilibria in this setting, we have made different propositions. Through which we have generated that these multiple solutions are indexed by the value of u_0. For different values of u_0 we get different values of all other variables i.e, $c(t)$, $K(t)$ and $T(t)$. For the numerical simulations we have discussed the effect of change in organizational development on foreign technology and the efficiency of utilization of foreign technology and we have seen that the organizational development is positively related to the foreign technology but it is negatively related to the efficiency of utilization of foreign technology. Also the transitional dynamics of foreign technology and the efficiency of utilization of foreign technology is discussed in the form of cross country analysis.

CONCLUSION

The thesis aims to present an endogenous growth model to investigate the impact of an organizational development on the economic growth. For this, we have formulated a two sector growth model where an organizational development impacts physical capital and foreign technology with the effect of externality in the production function. The organizational development impacts two sectors physical capital and foreign technology by effecting the growth rate of foreign technology $B(\theta)$ and the adjustment cost $G(\theta)$ respectively. The growth rate of foreign technology is positively related to the organizational development, higher the value of organizational development θ the higher will be the growth rate of foreign technology. Whereas, the adjustment cost accumulated in building up the organization is negatively related to the development of an organization. The more the organization is developed, the higher will be the value of θ and less resources will be wasted in the production of goods and services which will ultimately reduce the cost of production.

Furthermore, we have solved the model through the closed form solutions where the inverse of intertemporal elasticity of substitution σ is equal to the share of physical capital α for all the variables in the model (Xie, 1994). The term externality of foreign technology used in the production function plays an important role in our model. A one unit increase in the level of foreign technology produces an additional level of output. This results in the two interesting properties.

The first one where the effect of externality is less than the share of physical capital; this results in the unique equilibrium value of all the variables in the model. For such case we have generated different numerical simulations where the effect of change in the organizational development is observed on the consumption $c(t)$, foreign technology $T(t)$, stock of physical capital $K(t)$, efficiency of utilization of foreign technology $u(t)$ and the total production $Y(t)$. An increase in the level of organizational development will results an increase in the level of total consumption, foreign technology, stock of physical capital and the production. Whereas, the efficiency of utilization of foreign technology remains the same. These results coincide with the literature as well.

The second one, which results in the multiple equilibrium paths is the case where the share of physical capital is less than the effect of externality of foreign technology in the production function. Under this setting, all the state and co-state variables of the model converge to the different equilibrium values indicated by the value of control variable u_0. The change in the level of organizational development where the difference between the effect of externality and the share of physical capital is positive; it positively impacts the efficiency of utilization of foreign technology and negatively impacts the total stock of foreign technology.

This has been explained by help of cross country analysis. The country with lower endowments of physical capital and foreign technology can have higher steady state value as long as it has smaller u_0. This means that the country with lower endowments will overtake the richer country at some point in time. That is, the country with an initial higher endowments will start below but will finish first (Marquez and Ramon, 2005).

To conclude this, a well developed organization tends to increase the total stock of physical capital and the foreign technology which will hence accumulate the economic growth.

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Appendix A

Appendix A1

The steady state solution of three dynamical equations system can be found by setting (1.35), (1.36) and (1.37) equal to zero,

$$0 = A[1-G(\theta)]\left(\frac{\alpha}{\sigma}-1\right)u^{*1-\alpha}\psi^{*\alpha-1} + \chi^* - \frac{\rho}{\sigma} \tag{A1-1}$$

$$0 = A[1-G(\theta)]u^{*1-\alpha}\psi^{*\alpha-1} - \chi^* - \frac{1-\alpha+\gamma}{1-\alpha}a\overline{T}B(\theta)(1-u^*) \tag{A1-2}$$

$$0 = -\chi^* + \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u^* + \frac{a\overline{T}B(\theta)(1+\gamma-\alpha)}{\alpha} \tag{A1-3}$$

By plugging equation (A1-1) into equation (A1-2) we'll obtain,

$$A[1-G(\theta)]u^{*1-\alpha}\psi^{*\alpha-1} = \frac{\rho}{\alpha-\sigma} - \chi^*\frac{\sigma}{\alpha-\sigma} \tag{A1-4}$$

By substituting (A1-4) in (A1-2)we get,

$$0 = \frac{\rho}{\alpha-\sigma} - \chi^*\left[\frac{\sigma}{\alpha-\sigma}+1\right] - \frac{(1-\alpha+\gamma)}{(1-\alpha)}a\overline{T}B(\theta)(1-u^*) \tag{A1-5}$$

Solving the above equation for χ^* we get,

$$\chi^* = \frac{\rho}{\alpha} - \frac{(\alpha-\sigma)(1-\alpha+\gamma)}{\alpha(1-\alpha)}a\overline{T}B(\theta)(1-u^*) \tag{A1-6}$$

By substituting equation (A1-6) into equation (A1-3)

$$0 = \frac{(\alpha-\sigma)(1-\alpha+\gamma)}{\alpha(1-\alpha)}a\overline{T}B(\theta)(1-u^*) - \frac{\rho}{\alpha} + \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u^* + \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{\alpha} \tag{A1-7}$$

$$\begin{aligned}\frac{(\alpha-\sigma)(1-\alpha+\gamma)a\overline{T}B(\theta)u^* - (1-\alpha)a\overline{T}B(\theta)(\alpha-\gamma)u^*}{\alpha(1-\alpha)} &= \frac{-\rho(1-\alpha)}{\alpha(1-\alpha)} \\ &+ \frac{a\overline{T}B(\theta)(\alpha-\sigma)(1-\alpha+\gamma) + a\overline{T}B(\theta)(1-\alpha)(1-\alpha+\gamma)}{\alpha(1-\alpha)}\end{aligned} \tag{A1-8}$$

$$\begin{aligned}a\overline{T}B(\theta)u^*[\alpha\sigma-\gamma\sigma+\gamma-\sigma] &= -a\overline{T}B(\theta)\sigma + a\overline{T}B(\theta)\alpha\sigma - a\overline{T}B(\theta)\sigma\gamma + a\overline{T}B(\theta) - a\overline{T}B(\theta)\alpha \\ &+ a\overline{T}B(\theta)\gamma - \rho + \rho\alpha\end{aligned} \tag{A1-9}$$

Making u^* as a subject from above equation, we get

$$u^* = 1 - \frac{a\overline{T}B(\theta) - a\overline{T}B(\theta)\alpha - \rho + \rho\alpha}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma) - \gamma]} \tag{A1-10}$$

Thus,

$$u^* = 1 - \frac{(1-\alpha)(a\overline{T}B(\theta) - \rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma) - \gamma]} \tag{A1-11}$$

Now by substituting the value of u^* from equation (A1-11) to equation (A1-6) we get,

$$\chi^* = \frac{\rho}{\alpha} - \frac{(\alpha-\sigma)(1-\alpha+\gamma)}{\alpha(1-\alpha)} a\overline{T}B(\theta)\left(1 - [1 - \frac{(1-\alpha)(a\overline{T}B(\theta) - \rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma) - \gamma]}]\right) \tag{A1-12}$$

Thus,

$$\chi^* = \frac{a\overline{T}B(\theta)}{\alpha} + \frac{(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta) - \rho)}{\alpha[\sigma(1-\alpha+\gamma) - \gamma]} \tag{A1-13}$$

Putting the value of u^* and χ^* from equation (A1-11) and (A1-13) into equation (A1-1), we get

$$\begin{aligned} 0 = A[1-G(\theta)]\left(\frac{\alpha}{\sigma} - 1\right)\left[1 - \frac{(1-\alpha)(a\overline{T}B(\theta) - \rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma) - \gamma]}\right]^{1-\alpha} \psi^{*\alpha-1} + \frac{a\overline{T}B(\theta)}{\alpha} \\ + \frac{(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta) - \rho)}{\alpha[\sigma(1-\alpha+\gamma) - \gamma]} - \frac{\rho}{\sigma} \end{aligned} \tag{A1-14}$$

Solving the above equation for ψ^* we get

$$\psi^* = \left[\frac{\sigma(1-\alpha+\gamma)a\overline{T}B(\theta) - \gamma\rho}{A[1-G(\theta)]\alpha[(1-\alpha+\gamma)\sigma - \gamma]}\right]^{\frac{1}{\alpha-1}} u \tag{A1-15}$$

Appendix A2

When $u^* < 1$

$$1 - \frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]} < 1 \tag{A2-1}$$

$$-\frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]} < 0 \tag{A2-2}$$

$$\frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]} > 0 \tag{A2-3}$$

which is true provided $a\overline{T}B(\theta)-\rho$ and $\sigma(1-\alpha+\gamma)-\gamma$ have same sign
Case 1: When $u^* > 0$ provided $\left[a\overline{T}B(\theta)-\rho > 0, \sigma(1-\alpha+\gamma)-\gamma > 0\right]$

$$1 - \frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]} > 0 \tag{A2-4}$$

$$a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma] - (1-\alpha)(a\overline{T}B(\theta)-\rho) > 0 \tag{A2-5}$$

$$a\overline{T}B(\theta)\sigma(1-\alpha+\gamma) - a\overline{T}B(\theta)\gamma > (1-\alpha)(a\overline{T}B(\theta)-\rho) \tag{A2-6}$$

$$\sigma > \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} \tag{A2-7}$$

$$\sigma > 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} \tag{A2-8}$$

where $\sigma_m = 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$
Case 2: When $u^* > 0$ provided $\left[a\overline{T}B(\theta)-\rho < 0, \sigma(1-\alpha+\gamma)-\gamma < 0\right]$

$$1 - \frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]} > 0 \tag{A2-9}$$

$$a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma] - (1-\alpha)(a\overline{T}B(\theta)-\rho) < 0 \tag{A2-10}$$

$$a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma] - (1-\alpha)(a\overline{T}B(\theta)-\rho) < 0 \tag{A2-11}$$

$$a\overline{T}B(\theta)\sigma(1-\alpha+\gamma) < (1-\alpha)(a\overline{T}B(\theta)-\rho) + \gamma a\overline{T}B(\theta) \tag{A2-12}$$

$$\sigma < \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} \tag{A2-13}$$

$$\sigma < 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} \tag{A2-14}$$

For $a\overline{T}B(\theta) > \rho$ implies $\sigma > 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$

For $a\overline{T}B(\theta) < \rho$ implies $\sigma < 1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$

But $\sigma > 0$ indicates that $1 - \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} > 0$

$1 > \frac{\rho(1-\alpha)}{a\overline{T}B(\theta)(1-\alpha+\gamma)}$

$\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{(1-\alpha)} > \rho$

where $\rho < \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{1-\alpha}$

and $\rho_m = \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{1-\alpha}$

Appendix A3

For $\chi > 0$

$\frac{a\overline{T}B(\theta)}{\alpha} + \frac{(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho)}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]} > 0$

For $\gamma > \alpha$, χ is positive as $a\overline{T}B(\theta) - \rho$ and $\sigma(1-\alpha+\gamma)-\gamma$ have same sign.

Case 1: When $\gamma < \alpha$ provided $\left[a\overline{T}B(\theta) - \rho > 0, \sigma(1-\alpha+\gamma) - \gamma > 0\right]$

$$a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma] > -(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho) \tag{A3-1}$$

$$a\overline{T}B(\theta)\sigma(1-\alpha+\gamma) > a\overline{T}B(\theta)\gamma + (1-\alpha)(\alpha-\gamma)(a\overline{T}B(\theta)-\rho) \tag{A3-2}$$

$$a\overline{T}B(\theta)\sigma(1-\alpha+\gamma) > a\overline{T}B(\theta)\gamma-(1-\alpha)(\alpha-\gamma)\rho+a\overline{T}B(\theta)(1-\alpha)(\alpha-\gamma) \tag{A3-3}$$

$$a\overline{T}B(\theta)\sigma(1-\alpha+\gamma) > \alpha a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho(1-\alpha)(\alpha-\gamma) \tag{A3-4}$$

$$\sigma > \alpha - \frac{\rho(1-\alpha)(\alpha-\gamma)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} \tag{A3-5}$$

Where, $\alpha - \frac{\rho(1-\alpha)(\alpha-\gamma)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} = \sigma_n$

For this case $\sigma_m > \sigma_n$

Case 2: When $\gamma < \alpha$ provided $\left[a\overline{T}B(\theta) - \rho < 0, \sigma(1-\alpha+\gamma) - \gamma < 0\right]$

$$\frac{a\overline{T}B(\theta)}{\alpha} + \frac{(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho)}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]} > 0 \tag{A3-6}$$

$$a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma] + (1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho) < 0 \tag{A3-7}$$

As $\sigma(1-\alpha+\gamma)-\gamma<0$

$$a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma] < -(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho) \tag{A3-8}$$

$$a\overline{T}B(\theta)\sigma(1-\alpha+\gamma) < a\overline{T}B(\theta)\gamma+(1-\alpha)(\gamma-\alpha)\rho-(1-\alpha)a\overline{T}B(\theta)(\gamma-\alpha) \tag{A3-9}$$

$$\sigma < \alpha - \frac{\rho(1-\alpha)(\alpha-\gamma)}{a\overline{T}B(\theta)(1-\alpha+\gamma)} \tag{A3-10}$$

For this case $\sigma_m < \sigma_n$

Appendix A4

The growth rates for c, K and T can be found with the aid of equations (1.23) and (1.24), which are as following

From equation (1.24)

$$\frac{\dot{T}}{T} = a\overline{T}B(\theta)(1-u) \tag{A4-1}$$

Plugging the value of u in above equation

$$\frac{\dot{T}}{T} = a\overline{T}B(\theta)\left[1-1+\frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{\overline{T}B(\theta)\sigma(1-\alpha+\gamma)-\gamma}\right] \tag{A4-2}$$

$$\frac{\dot{T}}{T} = \frac{a\overline{T}B(\theta)(1-\alpha)(a\overline{T}B(\theta)-\rho)}{a\overline{T}B(\theta)[\sigma(1-\alpha+\gamma)-\gamma]} \tag{A4-3}$$

Hence,

$$\frac{\dot{T}}{T} = \frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{\sigma(1-\alpha+\gamma)-\gamma} = g_T \tag{A4-4}$$

Now from equation (1.29)

$$\frac{\dot{c}}{c} = -\frac{1}{\sigma}\left[-A[1-G(\theta)]\alpha K^{\alpha-1}u^{1-\alpha}T^{1-\alpha+\gamma}+\rho\right] \tag{A4-5}$$

$$\frac{\dot{c}}{c} = -\frac{\rho}{\sigma} + A[1-G(\theta)]\alpha K^{\alpha-1}u^{1-\alpha}T^{1-\alpha+\gamma} \tag{A4-6}$$

$$\frac{\dot{c}}{c} = -\frac{\rho}{\sigma} + A[1-G(\theta)]\alpha u^{1-\alpha}(\psi)^{1-\alpha} \tag{A4-7}$$

$$\frac{\dot{c}}{c} = \frac{-\rho\alpha(1-\alpha+\gamma)+\rho\gamma+\sigma a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho\gamma}{[\sigma(1-\alpha+\gamma)\sigma-\gamma]} \tag{A4-8}$$

Hence,

$$\frac{\dot{c}}{c} = \frac{(1-\alpha+\gamma)(a\overline{T}B(\theta)-\rho)}{\sigma(1-\alpha+\gamma)-\gamma} = g_c \tag{A4-9}$$

From equation (1.23) we have,

$$\frac{\dot{K}}{K} = A[1-G(\theta)]K^{\alpha-1}u^{1-\alpha}T^{1-\alpha+\gamma} - \frac{c}{K} \tag{A4-10}$$

$$\frac{\dot{K}}{K} = A[1-G(\theta)]u^{1-\alpha}\psi^{1-\alpha} - \chi \tag{A4-11}$$

$$\begin{aligned}\frac{\dot{K}}{K} = {} & \frac{a\overline{T}B(\theta)\sigma(1-\alpha+\gamma)-\rho\gamma}{\alpha\sigma(1-\alpha+\gamma)-\alpha\gamma} - \frac{a\overline{T}B(\theta)}{\alpha} \\ & - \frac{(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\rho)}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]}\end{aligned} \tag{A4-12}$$

$$\frac{\dot{K}}{K} = \frac{(a\overline{T}B(\theta)-\rho)\gamma-(1-\alpha)(\gamma-\alpha)(a\overline{T}B(\theta)-\gamma)}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]} \tag{A4-13}$$

$$\frac{\dot{K}}{K} = \frac{(a\overline{T}B(\theta)-\rho)[\gamma-(1-\alpha)(\gamma-\alpha)]}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]} \tag{A4-14}$$

$$\frac{\dot{K}}{K} = \frac{(a\overline{T}B(\theta)-\rho)[\gamma-\gamma+\alpha+(\gamma-\alpha)]}{\alpha[\sigma(1-\alpha+\gamma)-\gamma]} \tag{A4-15}$$

$$\frac{\dot{K}}{K} = \frac{(a\overline{T}B(\theta)-\rho)(1-\alpha+\gamma)}{\sigma(1-\alpha+\gamma)-\gamma} = g_K \tag{A4-16}$$

Thus,

$$g_c = g_K = \frac{(a\overline{T}B(\theta)-\rho)(1-\alpha+\gamma)}{\sigma(1-\alpha+\gamma)-\gamma} \tag{A4-17}$$

$$g_T = \frac{(1-\alpha)(a\overline{T}B(\theta)-\rho)}{\sigma(1-\alpha+\gamma)-\gamma} \tag{A4-18}$$

$g_c = g_K > 0$ and $g_T > 0$

Provided $a\overline{T}B(\theta)-\rho$ and $\sigma(1-\alpha+\gamma)-\gamma$ have same sign.

Appendix B

Appendix B1

From (1.36), under the assumption $\sigma = \alpha$, we get

$$\frac{\dot{\chi}}{\chi} = \chi - \frac{\rho}{\alpha} \tag{B1-1}$$

Multiplying both sides by χ we have,

$$\dot{\chi} + \frac{\rho\,\chi}{\alpha} = \chi^2 \tag{B1-2}$$

which is Bernoulli's order differential equation (ODE)

By solving (B1-2) through Bernoulli's ODE technique, we get

$$\chi(t) = \frac{\alpha(\frac{\rho}{\alpha})e^{-(\frac{\rho}{\alpha})t}}{\alpha e^{-(\frac{\rho}{\alpha})t} + c_1\alpha} \tag{B1-3}$$

$$\chi(t) = \frac{(\frac{\rho}{\alpha})}{1 + c_1 e^{(\frac{\rho}{\alpha})}t} \tag{B1-4}$$

$$\chi_0 = \frac{(\frac{\rho}{\alpha})}{1 + c_1} \tag{B1-5}$$

By solving the above equation for c_1, we get

$$c_1 = \frac{(\frac{\rho}{\alpha})}{\chi_0} - 1 \tag{B1-6}$$

Putting the value of c_1 in (B1-6), we have

$$\chi(t) = \frac{(\frac{\rho}{\alpha})\chi_0}{\chi_0 + ((\frac{\rho}{\alpha}) - \chi_0)e^{(\frac{\rho}{\alpha})t}} \tag{B1-7}$$

Since $\chi = \frac{c}{K}$ and $c = \lambda^{-\frac{1}{\sigma}}$ Then, (1.25) can be written as:

$$\lim_{t\to\infty} \lambda K e^{-\rho t} = \lim_{t\to\infty} (\lambda c)(\frac{K}{c})e^{-\rho t} \tag{B1-8}$$

$$\lim_{t\to\infty} \lambda K e^{-\rho t} = \lim_{t\to\infty} \lambda.\lambda^{-\frac{1}{\alpha}} \left[\frac{\chi_0 + ((\frac{\rho}{\alpha}) - \chi_0)e^{-(\frac{\rho}{\alpha})t}}{(\frac{\rho}{\alpha})\chi^0}\right] e^{-\rho t} \tag{B1-9}$$

$$\lim_{t\to\infty} \lambda K e^{-\rho t} = \lim_{t\to\infty} \frac{\lambda^{\frac{\alpha-1}{\alpha}}}{(\frac{\rho}{\alpha})} e^{-\rho t} + \lim_{t\to\infty} \lambda^{\frac{\alpha-1}{\alpha}} \frac{((\frac{\rho}{\alpha}) - \chi_0)}{(\frac{\rho}{\alpha})\chi_0} e^{(\frac{\rho}{\alpha} - \rho)t} \tag{B1-10}$$

Which is equation (2.3) in the main text of thesis.

Appendix B2

Since χ is known in the Proposition 1 so we can calculate u. From (1.38) we can replace $\chi = \frac{\rho}{\alpha}$, given as

$$\frac{\dot{u}}{u} = -\frac{\rho}{\alpha} + \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u + \frac{1}{\alpha}a\overline{T}B(\theta)(1-\alpha+\gamma) \tag{B2-1}$$

$$\frac{\dot{u}}{u} = \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u + \frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho}{\alpha} \tag{B2-2}$$

Multiplying both sides by u, we have

$$\dot{u} + \frac{\rho - a\overline{T}B(\theta)(1-\alpha+\gamma)}{\alpha}u = \frac{a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}u^2 \tag{B2-3}$$

which is a Bernoulli's differential equation in u. Taking $\frac{\dot{u}}{u} = 0$ to compute u^*. Hence,

$$u^* = \frac{\rho - a\overline{T}B(\theta)(1-\alpha+\gamma)}{a\overline{T}B(\theta)(\alpha-\gamma)} \tag{B2-4}$$

Solving (B2-3) through Bernoulli's differential technique, we can have $u(t)$ as following:

$$u(t) = \frac{[\frac{\rho}{\alpha} - a\overline{T}B(\theta)\frac{1-\alpha+\gamma}{\alpha}]u_0}{[\frac{\rho}{\alpha} - \frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma) + \frac{a\overline{T}B(\theta)}{\alpha}(\gamma-\alpha)u_0]e^{\frac{\rho}{\alpha} - a\overline{T}B(\theta)(\frac{(1-\alpha+\gamma)}{\alpha})t} - \frac{a\overline{T}B(\theta)}{\alpha}(\gamma-\alpha)u_0} \tag{B2-5}$$

We can re-write the above equation as following:

$$u(t) = \frac{u^* u_0 N(\alpha-\gamma)}{[u^* u_0 a\overline{T}B(\theta)(\alpha-\gamma) - a\overline{T}B(\theta)(\alpha-\gamma)u_0]e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + a\overline{T}B(\theta)(\alpha-\gamma)u_0} \tag{B2-6}$$

Taking $a\overline{T}B(\theta)(\alpha-\gamma)$ as common, we have

$$u(t) = \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0} \tag{B2-7}$$

$$\lim_{t\to\infty} u(t) = \lim_{t\to\infty} \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0} \tag{B2-8}$$

$\lim_{t\to\infty} u(t) = u^* = u_0$ provided $a\overline{T}B(\theta)(\alpha-\gamma)[u^*-u_0] = 0$

Thus, $u^* = u_0 = 1 - \frac{a\overline{T}B(\theta)-\rho}{a\overline{T}B(\theta)(\alpha-\gamma)}$

Provided $u < 1$, then

$\frac{\rho - a\overline{T}B(\theta)(1-\alpha+\gamma)}{a\overline{T}B(\theta)(\alpha-\gamma)} < 1$

$\rho - a\overline{T}B(\theta)(1-\alpha+\gamma) - a\overline{T}B(\theta)(\alpha-\gamma) < 0$

$\rho - a\overline{T}B(\theta) + a\overline{T}B(\theta)(\alpha-\gamma) - a\overline{T}B(\theta)(\alpha-\gamma) < 0$

$\rho - a\overline{T}B(\theta) < 0$ or $a\overline{T}B(\theta) - \rho > 0$

Hence it is proved that the value of u lies between $0 < u < 1$ if and only if $\alpha > \gamma$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) < \rho < a\overline{T}B(\theta)$.

Provided $u > 0$, then

$1 - \frac{a\overline{T}B(\theta)-\rho}{a\overline{T}B(\theta)(\alpha-\gamma)} > 0$

$1 > \frac{a\overline{T}B(\theta)-\rho}{a\overline{T}B(\theta)(\alpha-\gamma)}$

$\rho > a\overline{T}B(\theta)[1-\alpha+\gamma]$

Appendix B3

From (1.24) we have $\dot{T} = a\overline{T}B(\theta)(1-u^*)T$

$$T(t) = T_0 e^{a\overline{T}B(\theta)(1-u^*)t} \tag{B3-1}$$

Put $u^* = 1 - \frac{a\overline{T}B(\theta)-\rho}{a\overline{T}B(\theta)(\alpha-\gamma)}$ in above equation

$$T(t) = T_0 e^{(\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma})t} \tag{B3-2}$$

Consequently, from (1.26) the transversality condition for T(t) gives:

$$\lim_{t\to\infty} T\mu e^{-\rho t} = \mu_0 e^{(\rho - a\overline{T}B(\theta))t} e^{-\rho t} T_0 e^{(\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma})t} \tag{B3-3}$$

$$\lim_{t\to\infty} T\mu e^{-\rho t} = \lim_{t\to\infty} \mu_0 e^{\frac{(a\overline{T}B(\theta)-\rho) - a\overline{T}B(\theta)(\alpha-\rho)}{\alpha-\gamma}t} \tag{B3-4}$$

$\frac{\dot{T}}{T} = \frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma}$

$\lim_{t\to\infty} \frac{\dot{T}}{T} = \frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma} > 0$ Provided, $(a\overline{T}B(\theta)-\rho)$ and $(\alpha-\gamma)$ have same signs, i.e, $(\alpha-\gamma) > 0$ and $(a\overline{T}B(\theta)-\rho) > 0$.

Appendix B4

From (1.23) we have

$$\dot{K} = A[1 - G(\theta)]u^{1-\alpha}T^{1-\alpha+\gamma}K^{\alpha} - c \tag{B4-1}$$

Using $c = \frac{\rho}{\alpha}K$

$$\dot{K} + \frac{\rho}{\alpha}K = A[1 - G(\theta)]u^{*1-\alpha}T_0^{1-\alpha+\gamma}e^{\frac{a\overline{T}B(\theta)-\rho}{\alpha-\gamma}(1-\alpha+\gamma)t}K^{\alpha} \tag{B4-2}$$

Solving the above equation through Bernoulli's differential technique, we get

$$K(t) = e^{-\frac{\rho}{\alpha}t}\left[K_0^{1-\alpha} + \phi\left(e^{\frac{(a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma)t}{\alpha(\alpha-\gamma)}} - 1\right)\right]^{\frac{1}{1-\alpha}} \tag{B4-3}$$

Where,

$$\phi = \frac{\alpha(\alpha-\gamma)(1-\alpha)A[1-G(\theta)]u^{*1-\alpha}T_0^{1-\alpha+\gamma}}{a\overline{T}B(\theta)\alpha(1-\alpha+\gamma) - \rho\gamma} \tag{B4-4}$$

Thus, $K(t) > 0$ provided $\frac{a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma-\rho(\alpha-\gamma)}{\alpha(\alpha-\gamma)} < 0$

$a\overline{T}B(\theta)\alpha(1-\alpha+\gamma) - \rho\gamma - \rho\alpha + \rho\gamma < 0$

$a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho < 0$

or,

$\rho - a\overline{T}B(\theta)(1-\alpha+\gamma) > 0$

Next, we will verify for the transversality conditions

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty} K(t)[\frac{\rho}{\alpha}]^{-\alpha}K^{-\alpha}e^{-\rho t} \tag{B4-5}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty}[\frac{\rho}{\alpha}]^{-\alpha}K^{1-\alpha}e^{-\rho t} \tag{B4-6}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty}[\frac{\rho}{\alpha}]^{-\alpha}[K_0^{1-\alpha} + \phi(e^{\frac{a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma}{\alpha(\alpha-\gamma)}t} - 1)]e^{-\frac{\rho}{\alpha}(1-\alpha)t}e^{-\rho t} \tag{B4-7}$$

The growth rate for K is

$$g_k = \frac{\left(\Phi\,\alpha\,\left(a\overline{T}B\left(\theta\right)-\rho\right)\left(1-\alpha+\gamma\right) - \mathrm{e}^{-\frac{\left(a\overline{T}B(\theta)\alpha\,(1-\alpha+\gamma)-\rho\,\gamma\right)t}{\alpha\,(-\alpha+\gamma)}}\rho\,\left(-1+\alpha\right)\left({K_0}^{1-\alpha}-\Phi\right)\left(-\alpha+\gamma\right)\right)}{\alpha\,\left(-1+\alpha\right)\left(-\alpha+\gamma\right)\left(\Phi\,\mathrm{e}^{-\frac{\left(a\overline{T}B(\theta)\alpha\,(1-\alpha+\gamma)-\rho\,\gamma\right)t}{\alpha\,(-\alpha+\gamma)}} + {K_0}^{1-\alpha} - \Phi\right)}$$

(B4-8)

Thus $g_k > 0$

Provided $\frac{a\overline{T}B(\theta)\alpha(1-\alpha+\gamma)-\rho\gamma}{\alpha(\alpha-\gamma)} > 0$

or,

$a\overline{T}B(\theta)(1-\alpha+\gamma) > \frac{\rho}{\alpha}\gamma$

Appendix C

Appendix C1

From equation (1.24)

$$\frac{\dot{T}}{T} = a\overline{T}B(\theta)[1 - \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0}] \tag{C1-1}$$

$$\frac{dT}{T} = a\overline{T}B(\theta)[1 - \frac{u^* u_0}{(u^* - u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t} + u_0}]dt \tag{C1-2}$$

$$T = T_0[\frac{u_0}{u^*} - (\frac{u_0}{u^*} - 1)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^* t}]^{\frac{\alpha}{\alpha-\gamma}} e^{(1-u^*)a\overline{T}B(\theta)t} \tag{C1-3}$$

$$T = T_0[\frac{u_0}{u^*}e^{-u^* a\overline{T}B(\theta)\frac{\alpha-\gamma}{\alpha}t} - (\frac{u_0}{u^*} - 1)]^{\frac{\alpha}{\alpha-\gamma}} e^{a\overline{T}B(\theta)t} \tag{C1-4}$$

Hence,

$$T = T_0[1 - \frac{u_0}{u^*} + \frac{u_0}{u^*}e^{-\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}]^{\frac{\alpha}{\alpha-\gamma}} e^{a\overline{T}B(\theta)t} \tag{C1-5}$$

Alternatively $u(t)$ can be expressed as

Let

$$F(t) = \frac{u^* - u_0 + u_0 e^{-\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}}{u^*} \tag{C1-6}$$

$$\frac{e^{\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}F(t)}{u_0} = \frac{e^{\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}(u^* - u_0) + u_0}{u^* u_0} \tag{C1-7}$$

$$\frac{e^{\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}F(t)}{u_0} = \frac{1}{u(t)} \tag{C1-8}$$

$$u(t) = \frac{u_0}{F(t)}e^{\frac{u^* a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t} \tag{C1-9}$$

Consequently the transversality condition from (1.26) for $T(t)$ gives

$$\lim_{t\to\infty} T(t)u(t)e^{-\rho t} = \lim_{t\to\infty} T_0 F(t)^{\frac{\alpha}{\alpha-\gamma}} e^{(a\overline{T}B(\theta)-\rho)t}[u_0 e^{(\rho - a\overline{T}B(\theta))t}] \tag{C1-10}$$

$$\lim_{t\to\infty} T(t)u(t)e^{-\rho t} = u_0 T_0 \lim_{t\to\infty} F(t)^{\frac{\alpha}{\alpha-\gamma}} \tag{C1-11}$$

$$\lim_{t\to\infty} T(t)u(t)e^{-\rho t} = u_0 T_0 [\infty]^{\frac{\alpha}{\alpha-\gamma}} \tag{C1-12}$$

$$\lim_{t\to\infty} T(t)u(t)e^{-\rho t} = u_0T_0[0] = 0 \tag{C1-13}$$

provided $\gamma > \alpha$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) - \rho > 0$.

Note that,

$\lim_{t\to\infty} F(t) = \frac{u^*-u_0}{u^*} + \frac{u_0}{u^*}e^{\infty} = \infty$

Appendix C2

From equation (1.24)

$$\frac{\dot{T}}{T} = a\overline{T}B(\theta)[1 - \frac{u^*u_0}{(u^*-u_0)e^{\frac{a\overline{T}B(\theta)}{\alpha}(\alpha-\gamma)u^*t} + u_0}] \tag{C2-1}$$

Using the value of u from (3.2) and T from (3.1) and $c = \frac{\rho}{\alpha}K$, equation (1.32) is given as

$$\dot{K} + c = Au^{1-\alpha}T^{1-\alpha+\gamma}K^{\alpha} \tag{C2-2}$$

$$\dot{K} + \frac{\rho}{\alpha}K = A[1-G(\theta)][\frac{u_0}{F}e^{\frac{u^*a\overline{T}B(\theta)(\alpha-\gamma)}{\alpha}t}]^{1-\alpha}[T_0F^{\frac{\alpha}{\alpha-\gamma}}e^{a\overline{T}B(\theta)t}]^{1-\alpha+\gamma}K^{\alpha} \tag{C2-3}$$

$$\dot{K} + \frac{\rho}{\alpha}K = A[1-G(\theta)]u_0^{1-\alpha}T_0^{1-\alpha+\gamma}F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t-(1-\alpha)\frac{\rho}{\alpha}t}K_0^{\alpha} \tag{C2-4}$$

where, $z = K^{1-\alpha}, \dot{z} = (1-\alpha)K^{-\alpha}\dot{K}$

$$K^{-\alpha}\dot{K} + \frac{\rho}{\alpha}K^{1-\alpha} = A[1-G(\theta)]u_0^{1-\alpha}T_0^{1-\alpha+\gamma}F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t}e^{-(1-\alpha)\frac{\rho}{\alpha}t} \tag{C2-5}$$

$$\frac{1}{1-\alpha}\dot{z} + \frac{\rho}{\alpha}z = A[1-G(\theta)]u_0^{1-\alpha}T_0^{1-\alpha+\gamma}F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t}e^{-(1-\alpha)\frac{\rho}{\alpha}t} \tag{C2-6}$$

$$\dot{z} + \frac{\rho}{\alpha}(1-\alpha)z = A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t}e^{-(1-\alpha)\frac{\rho}{\alpha}t} \tag{C2-7}$$

Now, Integrating Factor is $e^{\frac{\rho}{\alpha}(1-\alpha)t}$

$$\frac{d}{dt}[ze^{\frac{\rho}{\alpha}(1-\alpha)t}] = A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} \tag{C2-8}$$

$$ze^{\frac{\rho}{\alpha}(1-\alpha)t} = A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}\int F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t}dt + c_1 \tag{C2-9}$$

$$K^{1-\alpha} = A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t)e^{-\frac{\rho}{\alpha}(1-\alpha)t} + c_1e^{-\frac{\rho}{\alpha}(1-\alpha)t} \quad \text{(C2-10)}$$

$$K_0^{1-\alpha} = 0 + c_1 \quad \text{(C2-11)}$$

$$K^{1-\alpha} = [K_0^{1-\alpha} + A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t)]e^{-\frac{\rho}{\alpha}(1-\alpha)t} \quad \text{(C2-12)}$$

$$K = [K_0^{1-\alpha} + A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t)]^{\frac{1}{1-\alpha}}e^{-\frac{\rho}{\alpha}t} \quad \text{(C2-13)}$$

where,

$$Z(t) = \int F^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t}dt \quad \text{(C2-14)}$$

$$F(t)^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} = [F(t)e^{\frac{a\overline{T}B(\theta)}{\alpha\gamma}(1-\alpha+\gamma)(\alpha-\gamma)t}]^{\frac{\gamma}{\alpha-\gamma}} \quad \text{(C2-15)}$$

Applying limit on both sides

$$\lim_{t\to\infty} F(t)^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} = \lim_{t\to\infty}[(\frac{u^*-u_0}{u^*})e^{\frac{a\overline{T}B(\theta)}{\alpha\gamma}(1-\alpha+\gamma)(\alpha-\gamma)t} + \frac{u_0}{u^*} e^{-u^*a\overline{T}B(\theta)\frac{(\alpha-\gamma)}{\alpha}}e^{\frac{a\overline{T}B(\theta)}{\alpha\gamma}(1-\alpha+\gamma)(\alpha-\gamma)}]^{\frac{\gamma}{\alpha-\gamma}} \quad \text{(C2-16)}$$

$$\lim_{t\to\infty} F(t)^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} = \lim_{t\to\infty}\frac{u_0}{u^*}e^{(\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{\gamma}-\frac{\rho}{\alpha})t} \quad \text{(C2-17)}$$

This approaches to zero provided, $\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)}{\gamma} - \frac{\rho}{\alpha} < 0$ Thus,

$$\lim_{t\to\infty} F(t)^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} = [0]^{\frac{\gamma}{\alpha-\gamma}} = (\frac{1}{0})^{\frac{\gamma}{\alpha-\gamma}} \quad \text{(C2-18)}$$

As $\alpha-\gamma<0$, the above equation can be re-written as,

$$\lim_{t\to\infty} F(t)^{\frac{\gamma}{\alpha-\gamma}}e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} = \infty \quad \text{(C2-19)}$$

$\lim_{t\to\infty} Z(t) = \infty$ provided, $\alpha-\gamma<0$ and $a\overline{T}B(\theta)(1-\alpha+\gamma) < \frac{\gamma\rho}{\alpha}$

Now we will check for the transversality condition of $K(t)$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty}[K(t)][(\frac{c_0}{K_0})^{-\alpha}(K(t))^{-\alpha}]e^{-\rho t} \quad \text{(C2-20)}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty}(\frac{c_0}{K_0})^{-\alpha}K(t)^{1-\alpha}e^(-\rho t) \quad \text{(C2-21)}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty}(\frac{c_0}{K_0})^{-\alpha}[K_0^{1-\alpha}+A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t)]e^{-\frac{\rho}{\alpha}(1-\alpha)t}e^{-\rho t}$$

(C2-22)

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty} \frac{(\frac{c_0}{K_0})^{-\alpha}[K_0^{1-\alpha} + A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}Z(t)]}{e^{\frac{\rho}{\alpha}t}} \tag{C2-23}$$

Differentiate with respect to $'t'$ we have,

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \lim_{t\to\infty} (\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma} \frac{Z'(t)}{\frac{\rho}{\alpha}e^{\frac{\rho}{\alpha}t}} \tag{C2-24}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \frac{(\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}}{\frac{\rho}{\alpha}} \lim_{t\to\infty} \frac{Z'(t)}{e^{\frac{\rho}{\alpha}t}} \tag{C2-25}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \frac{(\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}}{\frac{\rho}{\alpha}} \lim_{t\to\infty} F^{\frac{\gamma}{\alpha-\gamma}} e^{\frac{a\overline{T}B(\theta)}{\alpha}(1-\alpha+\gamma)t} e^{-\frac{\rho}{\alpha}t} \tag{C2-26}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \frac{(\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}}{\frac{\rho}{\alpha}} \lim_{t\to\infty} F^{\frac{\gamma}{\alpha-\gamma}} e^{(\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho}{\alpha})t} \tag{C2-27}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \frac{(\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}}{\frac{\rho}{\alpha}} \lim_{t\to\infty} [F e^{-(\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho}{\alpha})(\frac{\gamma-\alpha}{\gamma})t}]^{-\frac{\gamma-\alpha}{\gamma}} \tag{C2-28}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \frac{(\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}}{\frac{\rho}{\alpha}} \lim_{t\to\infty} [(\frac{u^* - u_0}{u^*}) e^{-[(\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho}{\alpha})(\frac{\gamma-\alpha}{\gamma})]t} + \frac{u_0}{u^*} e^{(\frac{a\overline{T}B(\theta)(1-\alpha+\gamma)-\rho}{\alpha})t}]^{-\frac{\gamma}{\gamma-\alpha}} \tag{C2-29}$$

$$\lim_{t\to\infty} K(t)\lambda(t)e^{-\rho t} = \frac{(\frac{c_0}{K_0})^{-\alpha} A[1-G(\theta)](1-\alpha)u_0^{1-\alpha}T_0^{1-\alpha+\gamma}}{\frac{\rho}{\alpha}} [0+\infty]^{-\frac{\gamma}{\gamma-\alpha}} = 0 \tag{C2-30}$$