On the validity of the Capital Asset Pricing Model

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Abstract

One of the most important developments of modern finance is the Capital Asset Pricing Model (CAPM) of Sharpe, Lintner and Mossin. Although the model has been the subject of several academic papers, it is still exposed to theoretical and empirical criticisms.

The CAPM is based on Markowitz's (1959) mean variance analysis. Markowitz demonstrated that rational investors would hold assets, which offer the highest possible return for a given level of risk, or conversely assets with the minimum level of risk for a specific level of return.

Building on Markowitz's work, Sharpe and Lintner after making a number of assumptions, developed an equilibrium model of exchange showing the return of each asset as a function of the return on the market portfolio. This model and its underlying assumptions are reviewed in section 1. This model known as the Capital Asset Pricing Model has since been the focus of a number of empirical tests, and as shown in sections 3 and 5 the majority of these tests deny the validity of the model. However, as discussed in sections 4 and 6 these tests have not been free of criticism. Section 2 briefly presents a framework under which the empirical tests of the CAPM can be carried out. Section 7 provides a conclusion.

1. The CAPM and its assumptions

Sharpe and Lintner assumed that there are no transaction costs and no income taxes. Further, they assumed that assets are infinitely divisible and there are no restrictions to short selling and that investors can lend and borrow unlimited amounts at the risk free rate of interest. More importantly they assumed the homogeneity of expectations and that individuals hold mean variance efficient portfolios. Another implicit assumption of the CAPM is that all assets including human capital are marketable. Moreover the CAPM is essentially a single period model.

It is clear that these assumptions do not hold in the real world and thus, not surprisingly, the model's validity has been suspect from the outset.

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However, on closer examination the assumptions underlying the CAPM are not as stringent as they first appear to be.

Exactly the same results would obtain if short sales were disallowed. Since in equilibrium no investor sells any security short, prohibiting short selling will not change the equilibrium. More formally the derivative of the Langrangian with respect to each security will have a Kuhn-Tucker multiplier added to it, but since each security is contained in the market portfolio, the value of the multiplier will be zero and hence the solution will remain unaffected.

Further, Fama (1970) and Elton and Gruber (1974 and 1975) give a set of conditions under which the multi-period problem reduces to a single period CAPM, where all individuals maximise a single period utility function. The conditions are that firstly consumers act as if the one-period returns are not state dependent, i.e. the distribution of one-period returns on all the assets are known at the beginning of the period. Secondly, the consumption opportunities are not state dependent and lastly consumers' tastes are independent of future events. Fama further shows that given these conditions the derived one period utility is equivalent to a multi-period utility function given nonsatiation and risk aversion. However, it is argued by many that the above conditions are rather restrictive.

Merton (1973) has shown that a necessary and sufficient condition for individuals to behave as if they were single-period maximisers and for the equilibrium return relationship of CAPM to hold is that the investment opportunity set is constant. Furthermore, the main results of the model hold if income tax and capital gains taxes are of equal sizes.

If the assumption of riskless lending or borrowing is violated, then Black (1972) has shown that we still obtain a linear relationship between an asset's returns and its risk as measured by the covariance of the assets returns with the market. This model as distinct from the standard CAPM is known as the zero beta CAPM.

Thus, even though the assumptions underlying the Capital Asset Pricing Model are demanding and have been the basis for much of the criticism against the model, nevertheless these assumptions are not altogether inflexible. More importantly, the final test of the model is not how reasonable the assumptions underlying it seem to be, but rather how well the model conforms with reality. Indeed, many proponents of the CAPM argue that due to technological advances, capital markets operate as if these assumptions are satisfied.

Sharpe and Lintner, thus making a number of assumptions, extended Markowitz's mean variance framework to develop a relationship for expected returns, which more precisely is

$$E[R_i] = R_f + \beta_{im}(E[R_m] - R_f)$$

$$\beta_{im} = \frac{Cov[R_i, R_m]}{Var[R_m]}$$

where R_i = return on asset i

R_m = return on the market portfolio

R_f = return on the riskless asset

Thus the return on an asset depends linearly on β_{im} (hereafter simply beta) which is a measure of the covariance of the asset's return with that of the market. Intuitively, in a rational and competitive market investors diversify all systematic risk away and thus price assets according to their systematic or non-diversifiable risk. Thus the model invalidates the traditional role of standard deviation as a measure of risk. This is a natural result of the rational expectations hypothesis (applied to asset markets) because if, on the contrary, investors also took into account diversifiable risks, then over time competition will force them out of the market. If, on the contrary, the CAPM does not hold, then the rationality of the asset's markets will have to be reconsidered.

Black has derived a more general version of the CAPM, which holds in the absence of a riskless asset. For this *zero beta CAPM*, we have

$$E[R_i] = E[R_{om}] + \beta_{im}(E[R_m] - E[R_{om}])$$

$$\beta_{im} = \frac{Cov[R_i, R_m]}{Var[R_m]}$$

where R_{om} = return on the zero beta portfolio, i.e. the portfolio (lying on the portfolio frontier) which has a zero correlation with the market portfolio.

2. Framework for testing the validity of CAPM

The standard CAPM can also be written in terms of excess returns

$$E[Z_i] = \beta_{im}E[Z_m]$$

$$\beta_{im} = \frac{Cov[R_i, R_m]}{Var[R_m]}$$

where $Z_i = R_i - R_f$

$$Z_m = R_m - R_f$$

Empirical tests of the standard CAPM have focused on three testable implications, namely the intercept is zero, beta completely captures the cross sectional expected returns and that the market excess return is positive. In this section the focus will be on the first testable implication, i.e. the intercept is zero.

The excess return market model is:

$$Z_t = \alpha + \beta Z_{mt} + \varepsilon_t$$

$$E[\varepsilon_t] = 0, \qquad E[\varepsilon_t \varepsilon_t'] = \Sigma$$

$$E[Z_{mt}] = \mu_{m,} \quad E[(Z_{mt} - \mu_m)^2] = \sigma_m^2$$

$$Cov[Z_{mt}, \varepsilon_t] = 0$$

where Z_t = (Nx1) vector of excess returns for N assets

β = (Nx1) vector of betas

Z_{mt} = time period t market portfolio of excess return

α = (Nx1) vector of intercepts

ε_t = (Nx1) vector of disturbances

The implication of the standard CAPM is that the vector of intercepts is zero. If this is true then the market portfolio will be the 'tangency' portfolio.

Assuming that the returns are IID and are normally distributed, the maximum likelihood estimation technique can be used to estimate the parameters α and β. The probability density function (pdf) of excess returns conditional on the market excess return is given by

$$f(Z_t \mid Z_{mt}) = (2\pi)^{-N/2} |\Sigma|^{-1/2} \text{ x } \exp[-(1/2)(Z_t - \alpha - \beta Z_{mt})'\Sigma^{-1}(Z_t - \alpha - \beta Z_{mt})]$$

and given that the returns are IID the joint pdf is

$$f(Z_1,\ldots,Z_T \mid Z_{m1},\ldots,Z_{mT}) = \prod f(Z_t \mid Z_{mt})$$

Thus the log-likelihood function is

$$\Lambda(\alpha, \beta, \Sigma) = -(NT/2)\log(2\pi) - (T/2)\log|\Sigma| - (1/2)\textstyle\sum_t (Z_t - \alpha - \beta Z_{mt})'\Sigma^{-1}(Z_t - \alpha - \beta Z_{mt})$$

The first order conditions are:

$$\partial\Lambda/\partial\alpha = \Sigma^{-1}[\sum_t(Z_t - \alpha - \beta Z_{mt})] = 0$$

$$\partial\Lambda/\partial\beta = \Sigma^{-1}[\sum_t(Z_t - \alpha - \beta Z_{mt})Z_{mt}] = 0$$

$$\partial\Lambda/\partial\Sigma = -(T/2)\Sigma^{-1} + (1/2)\Sigma^{-1}[\sum_t(Z_t - \alpha - \beta Z_{mt})(Z_t - \alpha - \beta Z_{mt})']\Sigma^{-1} = 0$$

Solving the above FOCs we get estimates for α, β and Σ, and it should be noted that these estimates are the same as the ones obtained using OLS. However, compared to the OLS they have better large sample properties.

The Wald test can then be used to check whether the intercept is zero. The Wald test statistic is given by

$$W = \Lambda\,[\mathrm{Var}(\Lambda)]^{-1}\Lambda$$

which has a chi-square distribution with N degrees of freedom.[1]

3. Empirical Tests of the CAPM

Most of the early tests of the CAPM employed the methodology of first estimating betas using time series regression and then running a cross sectional regression using the estimated betas as explanatory variables to test the hypothesis implied by the CAPM.

Using this approach one of the first tests of the CAPM was conducted by Lintner, which is reproduced in Douglas (1968). Using data from 1954-1963, Lintner ran the following regression

$$R_t = \alpha + bR_{mt} + e_t$$

where R_t = (Nx1) vector of asset returns

R_{mt} = return on the market portfolio

b = (Nx1) vector of estimated betas

Lintner then ran the following second pass regression:

$$\bar{R} = a_1 + a_2 b + a_3 S_e^2 + \eta$$

where S_e^2 = (NxN) matrix of residual variance (i.e. the variance of e in the first pass regression).

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[1] Since the material in this section is quite standard I will not venture further into this but I have included a detailed discussion of the statistical framework to test the CAPM in the appendix.

The testable implications of the CAPM are that $\alpha_1 = R_f$; $\alpha_2 = (E[R_m] - R_f)$ and $\alpha_3 = 0$.

However, Lintner found that the actual values did not confirm with the theoretical values. α_1 was found to be much larger than R_f or even R_{om}, α_2 was found to be statistically significant but had a lower value than expected and α_3 was found to be statistically significant as well. Thus Lintner's results seem to be in contradiction to the Capital Asset Pricing Model.

Fama and MacBeth (1973) performed one classic test of the CAPM. They combined the time series and cross-sectional steps to investigate whether the risk premia of the factors in the second pass regression were non-zero.

Forming 20 portfolios of securities, they estimated betas from a time-series regression similar to Lintner's methodology. However, they then performed a cross-sectional regression for each month over the period 1935-1968. Their second pass regression was of the following form:

$$R_t = \gamma_{0t} + \gamma_{1t}\beta - \gamma_{2t}\beta^2 + \gamma_{3t}S_e + \eta_t$$

If the standard CAPM was true then we should have the following:

- $E[\gamma_{0t}] = R_f$
- $E[\gamma_{1t}] > 0$ as the market risk premium should be positive
- $E[\gamma_{2t}] = 0$ as the securities market line (SML) should be linear, i.e. the relationship between return and the relevant risk should be linear.
- $E[\gamma_{3t}] = 0$ as the residual risk should not affect asset returns.

All of the above should be true if the standard CAPM is to hold.

Fama and MacBeth found that γ_3 was statistically insignificant and its value remains very small over several subperiods. Thus, in contrast to Lintner, they find that residual risk has no effect on security returns. Miller and Scholes (1972) showed that residual risk would act as a proxy for risk if beta had a large sampling error. This fact might reconcile Lintner's and Fama and MacBeth's results, as the latter's estimate for beta had much less sampling error due to their use of asset portfolios.

Fama and MacBeth further found that γ_2 is not statistically different from zero. Moreover, they found that the estimated mean of γ_1 is positive as predicted by the model. They also find that γ_0 is statistically different from

zero. However, their intercept is much greater than the risk free rate and thus this would indicate that the standard CAPM might not hold.

Black, Jensen and Scholes (1972) performed another classic test of the Capital Asset Pricing Model employing time-series regression. They ran the following familiar time series regression:

$$Z_t = \alpha + \beta Z_{mt} + \varepsilon_t$$

As observed before, the intercept should be zero according to the CAPM. Black *et al.* used the return on portfolios of assets rather than individual securities. Time series regression using returns on individual assets may give biased estimates, as it is likely that the covariance between residuals may not equal zero. This is not generally true with portfolios as they utilise more data.

The results from the BJS time series regressions show that the intercept term is different from zero and in fact is time varying. They find that when $\beta<1$ the intercept is positive and that it is negative when $\beta>1$. Thus, the findings of Black *et al.* violate the CAPM.

Stambaugh (1982) employs a slightly different methodology. From the market model we have

$$R_t = \alpha + \beta(R_{mt}) + e_t$$

If the CAPM was true then the intercept in the above equation should be constrained and should in fact be

$$\alpha = \kappa(1 - \beta)$$

where $\kappa = R_f$ (under the Sharpe-Lintner CAPM)

or $\kappa = R_{om}$ (under the Black's version of CAPM)

Stambaugh then estimates the market model and using the Lagrange multiplier test finds evidence in support of Black's version of CAPM but finds no support for the standard CAPM.

Gibbons (1982) uses a similar method as the one used by Stambaugh but instead of the LM test uses a likelihood ratio test. He uses the fact that if the CAPM is true then the constrained market model should have the same explanatory power as the unconstrained model, but if the CAPM is invalid then the unconstrained model should have significantly more

explanatory power than the constrained model. Using this test, Gibbons rejects both the standard and the zero beta CAPM.

4. Possible Biases in tests of Asset Pricing Theory

Miller and Scholes (1972) in their paper "Rates of return in relation to risk" discuss the statistical problems inherent in all the empirical studies of the CAPM. They point out that the CAPM in time series form is

$$\overline{R_t} = R_{ft} + \beta\,(\overline{R_{mt}} - R_{ft})$$

or

$$\overline{R_t} = (1 - \beta)R_{ft} + \beta\overline{R_{mt}}$$

and thus if the riskless rate is non-stochastic then the CAPM can easily be tested by finding whether the intercept is significantly different from $(1 - \beta)R_{ft}$. However, if R_{ft} varies with time and moreover is correlated with R_{mt}, then we inevitably encounter the problem of omitted variable bias and thus the estimated betas will be biased.

Miller and Scholes, then using historical data find that R_{ft} and R_{mt} are negatively correlated. Intuitively, a rise in the interest rates is conducive to stock market declines. They then prove that if R_{ft} and R_{mt} are negatively correlated then this will lead to an upward bias in the intercept and further the slope will be biased downwards. This is in fact what many empirical studies find and thus the fact that many studies reject the CAPM does not imply that it does not hold.

Another factor that may bias the intercept upward and the slope downwards is the presence of heteroskedasticity. However, Miller and Scholes find no evidence of heteroskedasticity.

Miller and Scholes then go on to show the biases that one may encounter in the two stage regressions used by Lintner and Douglas and by Fama and MacBeth. The problem in this methodology is that estimated betas instead of the true betas are used in the second pass regressions and thus any error in the first stage is carried to the second stage. Miller and Scholes show that this 'errors-in-variables problem' will bias the intercept upward and the slope downwards.

However, this problem can be encountered by grouping assets into portfolios, by using instrumental variables or by the direct bias adjustment of Shanken *et al.*

Another possible problem in many tests of the CAPM arises due to it being a single-period model. Most tests of the CAPM use time series regression, which is only appropriate, if the risk premia and betas are stationary, which is unlikely to be true.

In his influential paper "A critique of asset pricing theory's tests", Roll (1977) shows that there has been no single unambiguous test of the CAPM. He points out that tests performed by using any portfolio other than the true market portfolio are not tests of the CAPM but are tests of whether the proxy portfolio is efficient or not. Intuitively, the true market portfolio includes all the risky assets including human capital while the proxy just contains a subset of all assets.

If we choose a portfolio, say m from the sample efficient frontier as a proxy for the market, then from efficient set mathematics we know that the mean return on any asset or portfolio j, will be a weighted average of the return on m and the return on the portfolio which has a zero correlation with m, i.e.

$$R_j \equiv (1 - \beta_j)R_{om} + \beta_j R_m)$$

where

$$\beta_j = \frac{\text{Cov}(R_j, R_m)}{\text{Var}(R_m)}$$

More generally, if A and B are any two sample efficient portfolios, then the mean return on asset j is given by

$$R_j \equiv (1 - \beta_j)R_A + \beta_j R_B \; \forall j$$

Conversely, if R is the mean vector of returns and β is the (Nx1) vector of slope coefficients obtained by regressing asset returns on the returns of some portfolio m, then we have

$$R = R_{om}\iota + (R_m - R_{om})\beta$$

where ι = vector of ones

The above relationship will hold iff R_m is ex-post efficient. Thus if m is not efficient then mean returns will not be linearly related to betas. Using this result from efficient set mathematics, Roll asserted that the only testable implication of the CAPM is that the market portfolio is mean-variance efficient. All other implications of the model, including the linearity of expected returns and beta follow from the efficiency of the market portfolio and thus are not independently testable.

Furthermore, for a given sample of mean returns, there always exist an infinite number of ex-post mean-variance portfolios. For these portfolios there will be an exact linear relationship between sample returns and sample betas. This linearity will hold whether or not the true market portfolio is efficient. Thus, the two-parameter asset pricing theory is not testable unless all assets are included in the sample.

In contradiction to the above argument, Fama and MacBeth in their paper incorrectly state that there are three testable implications, namely that the relationship between expected returns and beta is linear; that beta is a complete measure of risk; and that given risk averseness, higher return should be associated with higher risk, i.e. $E[R_m] - E[R_{om}] > 0$. Roll points out that if *m* is efficient then all the above implications are not independently testable and further asserts that the last inequality follows from the mathematical implication of the assumption about *m* rather than risk averseness per se.

Thus the only testable hypothesis concerning the zero beta CAPM is that the individuals prefer portfolios which are mean-variance efficient and that the market portfolio is ex-ante efficient.

On the contrary, the famous paper of Black, Jensen and Scholes does not even mention the possible efficiency of the market portfolio and concludes that the relationship between expected returns and beta is not linear. This conclusion is enough to prove that the proxy used by BJS does not lie on the sample efficient frontier. If on the other hand, the proxy had been on the efficient part of the frontier than BJS would have found a linear relationship between mean returns and beta. This is all in accordance with efficient set mathematics.

The relevant testable implications of the Sharpe-Lintner CAPM can be illustrated by means of figure 1. In the figure, m^* is the tangent portfolio. If *m* is used as the proxy, then the return on the asset is given by

$$R_j = R_z + (R_m - R_z)\beta_j \qquad (a)$$

On the other hand, if m^* is used as the proxy, then the return on the asset is given by

$$R_j = R_f + (R_m^* - R_f)\beta_j^* \qquad (b)$$

It should be noted that since efficient orthogonal portfolios are unique, β_z^* should be non-zero.

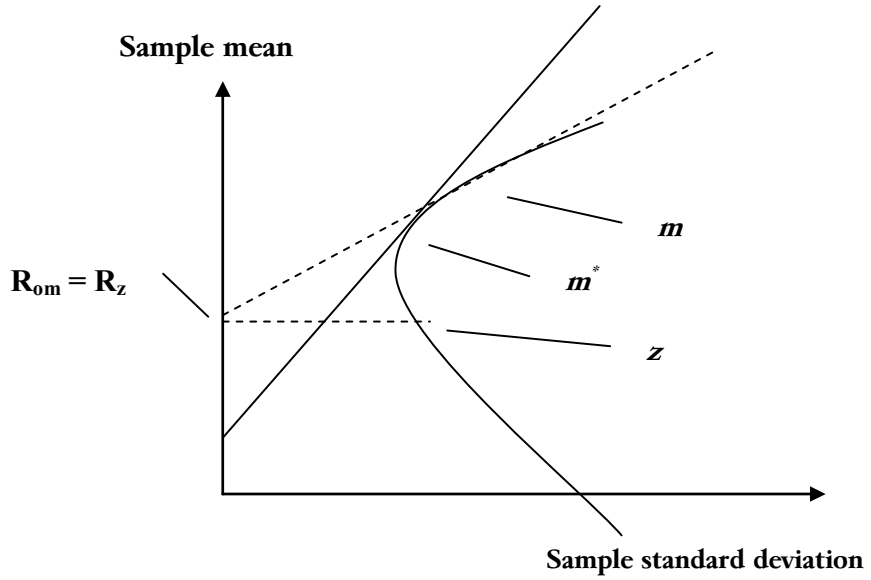

Figure 1

Since each individual will invest partly in the riskless asset and partly in the tangent portfolio m^*, thus the principle testable hypothesis of the Sharpe-Lintner CAPM is that the ex-ante efficient tangent portfolio is the market portfolio.

On the other hand, as already mentioned, BJS by using a market proxy estimated the following regression:

$$R_j - R_f = \alpha + \gamma\beta_j + \varepsilon_j \qquad (c)$$

They found that α was not only greater than zero but was also highly variable. Moreover, they found that γ was less than $R_m - R_f$. On the basis of these results, they rejected the standard CAPM.

However, Roll showed that unless BJS were successful in choosing the tangent portfolio m^* as their proxy, their results are actually in accord with the standard CAPM. Suppose, BJS chose m as their proxy, then substituting $j = z$ in (b), we have

$$R_z = R_f + (R_m^* - R_f)\beta_z^*$$

Substituting the above equation in (a) we get

$$R_j - R_f = \beta_z^*(R_m^* - R_f) + [R_m - R_f - \beta_z^*(R_m^* - R_f)]\beta_j \qquad (d)$$

Comparing (c) with (d) we see that if the Sharpe-Lintner CAPM is true than α should be equal to $\beta_z^*(R_m^* - R_f)$. Thus, since β_z^* is not equal to

0, α should in fact be not equal to zero and further since the return on the tangent portfolio m^* is a random variable, hence α should also be variable. Thus, it can be seen that the results of BJS are fully compatible with the standard CAPM!

Roll in his paper also shows that the proxy used by BJS was not even close to the tangent portfolio. However, even if BJS had found that the intercept was equal to zero, their result would not have invalidated the CAPM, simply because of the fact that they were not using the true market portfolio.

Fama and MacBeth in their study use the Fisher's Arithmetic index (an equally weighted portfolio of all the stocks in the NYSE) as their proxy. This portfolio is not even close to the value-weighted portfolio and thus should not have been used as a market proxy. Thus the conclusions of Fama and MacBeth are also not immune to suspicion.

It is clear that there will always exist a portfolio in the tangency position but it is not clear at all whether this portfolio is the value-weighted average of all assets, i.e. the market portfolio.

Furthermore, as shown by Roll, the situation is aggravated by the fact that both the Sharpe-Lintner CAPM and the Black's version of CAPM are liable to a type II error, i.e. likely to be rejected when they are true. This is true even if the proxy is highly correlated with the true market portfolio. Thus the efficiency or the inefficiency of the proxy does not imply anything about the efficiency of the true market portfolio.

It is not surprising therefore that Fama (1976) concluded that there has been no single unambiguous test of the CAPM.

5. Variables other than the market factor affecting stock returns

According to the CAPM, only market risk is priced, i.e. only beta affects returns and all other variables are irrelevant. However, there have been a number of empirical studies, which find that non-market factors have a significant affect on average returns.

Basu (1983) provides evidence that shares with high earnings yield (low price to earnings ratio) experience on average higher subsequent returns than shares with low earnings yield. Banz (1981) was the first to provide evidence of the 'size effect'; i.e. low market capitalisation firms have higher average returns compared to larger firms. Rosenberg, Reid and Lanstein (1985) show that firms with lower price-to-book ratios have higher mean returns.

A number of empirical studies also find support for the traditional use of the dividend-price ratio as a measure of expected stock returns. Sorenson and Williamson (1983) provide evidence that an increase in dividend yields raises average returns. Rozeff (1984), Flood, Hodrick and Kaplan (1987), Fama and French (1988) and Campbell and Shiller (1988) find similar results.

Merton (1973) has constructed a generalised intertemporal capital asset pricing model in which factors other than market uncertainty are priced. He models individuals as solving lifetime consumption decisions in a multi-period setting. After making a number of assumptions, Merton shows that the return on assets depend not only on the covariance of the asset with the market but also on its covariance with changes in the investment opportunity set. Hence, changes in interest rates, future income and relative prices will all influence returns. Intuitively, individuals will form portfolios to hedge themselves away from these risks. These actions of investor's will affect returns. According to this 'multi-beta CAPM', the return on securities will be affected by a number of indices apart from the market factor. Hence excess returns will be of the following form

$$E[R_i] - R_f = \beta_{im}(E[R_m] - R_f) + \beta_{iI1}(E[R_{I1}] - R_f) + \beta_{iI2}(E[R_{I2}] - R_f) + \ldots$$

This can also be interpreted as another form of the Arbitrage Pricing Theory (APT).

Elton and Gruber (1988), (1989) find that a five – factor APT model better explains expected returns compared to the classic CAPM model in the Japanese market. They find that in Japan, smaller stocks are associated with smaller betas and thus according to the CAPM they should give smaller mean returns. Yet, smaller stocks have higher expected returns than their larger counterparts. They also find that a multi-factor model is more useful in constructing hedge portfolios for futures and option trading.

Fama and French (1992) in an influential paper entitled, "The cross-section of expected stock returns", show that two easily measured variables, size and book-to-market equity combine to capture the cross-sectional variation in average stock returns associated with market beta, size, leverage, book-to-market, and earnings-to-price ratios.

Fama and French using data for non-financial firms conduct its asset-pricing tests using the Fama and MacBeth regressional approach. Their results from applying the FM regressions show that market beta clearly does not explain the average stock returns. The average slope from the regression of returns on beta alone is 0.15 per cent per month and it is only 0.46 standard errors from zero.

Furthermore, Fama and French point out that variables such as size, earning yield, leverage, and book-to-market are all scaled versions of a firm's stock price and thus some of them are redundant to explain returns. They show that of these variables only size and book-to-market equity explain cross-sectional average returns. Moreover, they find that when allowing for variations in beta that are unrelated to size, the relationship between beta and average return is flat. Hence, they naturally argue that the CAPM is dead.

6. Can the CAPM be saved?

The results of the previous section suggest that the CAPM is wrong and new alternatives have to be devised. However, as suggested earlier we cannot just discard the CAPM, until we can observe the true market portfolio.

One possible interpretation of the above findings is that the factors found to be significant in the above studies may actually be correlated with the true market portfolio.

Furthermore, Lo and MacKinlay (1990b) argue that biases relating to data-snooping may explain the observed deviations from the model. With hundreds of researchers examining the same data, some relationship between non-market factors and returns is bound to be significant. Lo and MacKinlay show that such biases, which are largely inevitable, may be immense especially in tests of the Sharpe-Lintner version of the CAPM.

Moreover, sample selection biases also exist in such studies as COMPUSTAT data exclude some stocks from the analysis. Kothari, Shanken, and Sloan (1995) argue that firms that have not been performing well are excluded. And since the failing stocks have a lower return and a high book-to-market ratio, thus the average returns of the included high book-to-market firms will be biased upwards. Kothari, et al. argue that this bias may explain the result found by Fama and French.

It is also claimed that measurement problems in estimating the CAPM may explain the observed 'size' effect. It is argued that the estimated betas for small firms are too low. If this is true then the CAPM will give a smaller expected return for small stocks and thus the difference between actual and expected returns will be large (and positive), even though it may actually be zero if there were no measurement errors associated with betas.

Christie and Hertzel (1981) point out that those firms, which become small also, become riskier but since beta is measured using historical returns, it does not capture this increased risk.

Further, Reinganum (1981) and Roll (1981) show that the beta estimated for small firms will be biased downward as they trade less frequently than do the larger firms.

Goetzmann and Jorion (1993) in their paper "Testing the predictive power of dividend yields" re-examine the ability of dividends to predict long-horizon stock returns. Using the bootstrap methodology as well as simulations to examine the distribution of test statistics under the null hypothesis of no forecasting ability, they find that the empirically observed statistics are well within the 95 per cent bounds of their simulated distributions. Overall, they find no statistical evidence to indicate that dividend yields affect stock returns.

They further argue that previous studies found a significant effect of dividend yields on returns as movements in prices dominate dividend yields. Thus the regressions suffer from biases as the right hand side variables are correlated with lagged dependent variables. Goetzmann and Jorion by using the bootstrap methodology explicitly incorporate the lagged price relation between returns and dividend yields and hence find no evidence of the significance of dividend yields.

Goetzmann and Jorion further claim that another reason for the results of the preceding studies is that both returns and dividend follow random walks and hence, following the result of Granger and Newbold (1974), the combination of the two series in a regression could result in spurious conclusions regarding explanatory power.

Roll and Ross (1994) in their recent paper point out again that a positive and exact cross-sectional relation between returns and betas must hold if the market index used is mean-variance efficient. If such a relationship were not found then this would suggest that the proxy used is ex ante inefficient. They further iterate that given that direct tests have rejected the mean-variance efficiency for many market proxies (e.g. Shanken (1985) and Zhou (1991)) it is not surprising that empirical studies find that the role of other variables in explaining cross-sectional returns is significant. However, what is surprising is the fact that some studies (e.g. Fama and French (1992)) find that the mean-beta relationship is virtually zero.

Roll and Ross then analyse where an index would have to be located to yield a specific relationship (including no or zero covariance) between a set of true betas and true expected returns. More specifically, they accomplish this by solving the following programme:

Minimise portfolio variance subject to the constraints:

(a) the portfolio's expected return is a given value;

(b) the portfolio weights sum to unity;

(c) and the cross-sectional regression of expected returns on betas has a particular slope.

Solving the above programme and defining k = the cross-sectional covariance of R and β, i.e. the numerator of the OLS slope from regressing individual expected returns on betas, they show that in the special case of $k = 0$, the market proxy (which gives a zero cross-sectional relationship between returns and beta) lies within a parabola which is inside the efficient frontier except for a tangency at the global minimum variance point. This is shown in figure 2 below.

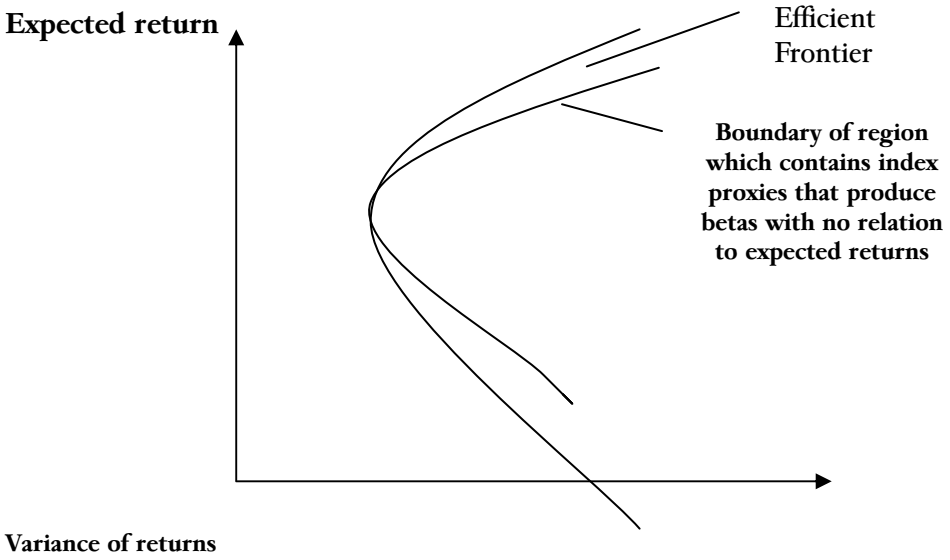

Figure 2

Roll and Ross then showed that assuming reasonable parameter values, a proxy which is just 22 basic points below the efficient frontier can give a cross-sectional mean-beta relationship that is actually zero. Hence, even small deviations of the market proxy index from the true market portfolio can give the wrong answer! Thus, a market proxy can be substantially inefficient and yet produce a strong cross-sectional relationship between expected returns and betas. Conversely, an index proxy can be quite close to the efficient frontier and still give a zero cross-sectional relation.

The situation is further aggravated by the presence of sampling error. With sampling error the power of cross-sectional tests is further reduced and therefore the probability of not rejecting a zero cross-sectional relation when the slope actually is not flat, may be quite high.

Thus a slope equal to or near zero tells us nothing about the validity of the SLB model.

The results of Roll and Ross are similar to those of Kandel and Stambaugh (1987), (1989). Kandel and Stambaugh after deriving the correlation between an arbitrary portfolio and the efficient portfolio, derive tests for the efficiency of an unknown index proxy with a given correlation with the true index. They thus argue that an unambiguous test of the CAPM can still be conducted ***conditional*** on the assumed correlation of the proxy with the true index.

Roll and Ross in their paper also show that depending on the econometric technique used, one can get a range of differing results with the same data. In particular they propound that the use of GLS instead of OLS always produces a positive cross-sectional relationship between expected returns and betas. This is true regardless of the efficiency of the proxy as long as the return on the proxy is greater than the return on the global minimum variance portfolio.

It is not surprising therefore that Amihud, Christensen, and Mendelson (1992) by using the superior technique of GLS, and by replicating Fama and French tests find that in contrast to the results of Fama and French, beta significantly affects expected returns.

Kandell and Stambaugh (1995) advocate the use of GLS as they show that by using GLS, R^2 increases as the proxy lies closer to the efficient frontier and vice versa. Thus the use of GLS can mitigate the extreme sensitivity of the cross-sectional results to deviations of the proxy from the true market portfolio. However, the problem with GLS is that the true parameters are unknown and hence the true covariance matrix of returns is also unknown. Furthermore, since the use of GLS results in almost every proxy producing a positive cross-sectional relation between mean returns and beta, hence unless other tests of efficiency are carried out, the results are by themselves of little significance.

7. Conclusion

Considering the arguments above, there is no doubt that it is not easy to give an unambiguous conclusion. On the one hand, there is strong

empirical evidence invalidating the CAPM and on the other hand it is clear that the empirical findings themselves are not sufficient to discard the CAPM.

Indeed, as noted by many authors including Fama and French in their recent article, "The CAPM is wanted, dead or alive", the empirical tests have been undermined by the inability to observe the true market portfolio. In effect, even though the 'synthetic' CAPM based on the proxy market index can be rejected, nevertheless it is virtually impossible to reject the original CAPM.

Nonetheless, since the true market portfolio cannot be observed it is fair to say that the CAPM is of little use for practical purposes. It cannot be used for estimating the cost of capital, to evaluate the performance of fund managers or as an aid in event-study analysis. This does not imply, however, that its substitute, the synthetic CAPM be used instead because as already seen there is a host of evidence against this form of the CAPM.

Given that we will have to work with the proxy index in the foreseeable future, thus for practical purposes, Merton's intertemporal CAPM or some form of the APT would have to be resorted to for the purpose of explaining expected stock returns.

References

Black, 1972, Capital market equilibrium with restricted borrowing, *Journal of Business*.

Black, Jensen, and Scholes, 1972, The capital asset pricing model: Some empirical tests, Studies in the Theory of Capital Markets.

Campbell, Lo, and MacKinlay, 1997, The Econometrics of Financial Markets, Princeton University Press.

Douglas, 1968, Risk in the equity markets: An empirical appraisal of market efficiency. University Microfilms, Inc., Ann Arbor, Mich.

Elton, and Gruber, 1975, Finance as a dynamic process, Prentice Hall.

Elton, and Gruber, 1993, Expectational data and Japanese stock prices, Japan and the World Economy 1.

Fama, 1970, Multi-period consumption-investment decision, *American Economic Review*.

Fama, and French, 1992, The cross-section of expected stock returns, *Journal of Finance*.

Fama, and French, 1996, The CAPM is wanted, dead or alive, *Journal of Finance*.

Fama, and MacBeth, 1974, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy*.

Gibbons, 1982, Multivariate tests of financial models: A new approach, *Journal of Financial Economics*.

Goetzmannn, and Jorion, 1993, Testing the predictive power of dividend yields, *Journal of Finance*.

Huang, and Litzenberger, 1988, Foundations for Financial Economics, Prentice Hall, Englewood Cliffs, New Jersey.

Kandel, and Stambaugh, 1995, Portfolio inefficiency and the cross-section of expected returns, *Journal of Finance*.

Kothari, Shanken, and Sloan, 1995, Another look at the cross-section of expected stock returns, *Journal of Finance*.

Lo, and MacKinlay, 1990, Data-snooping biases in tests of financial asset pricing models, *The Review of Financial Studies*.

Merton, 1973, An intertemporal capital asset pricing model, *Econometrica*.

Miller, and Scholes, 1972, Rates of return in relation to risk: A re-examination of some recent findings, *Studies in the Theory of Capital Markets*.

Reinganum, and Roll, 1981, A possible explanation of the small firm effect, *Journal of Finance*.

Roll, 1977, A critique of the asset pricing theory's tests; Part1: On past and potential testability of the theory, *Journal of Financial Economics*.

Roll, and Ross, 1994, On the cross-sectional relation between expected returns and betas, *Journal of Finance*.

Stambaugh, 1982, On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis, *Journal of Financial Economics*.